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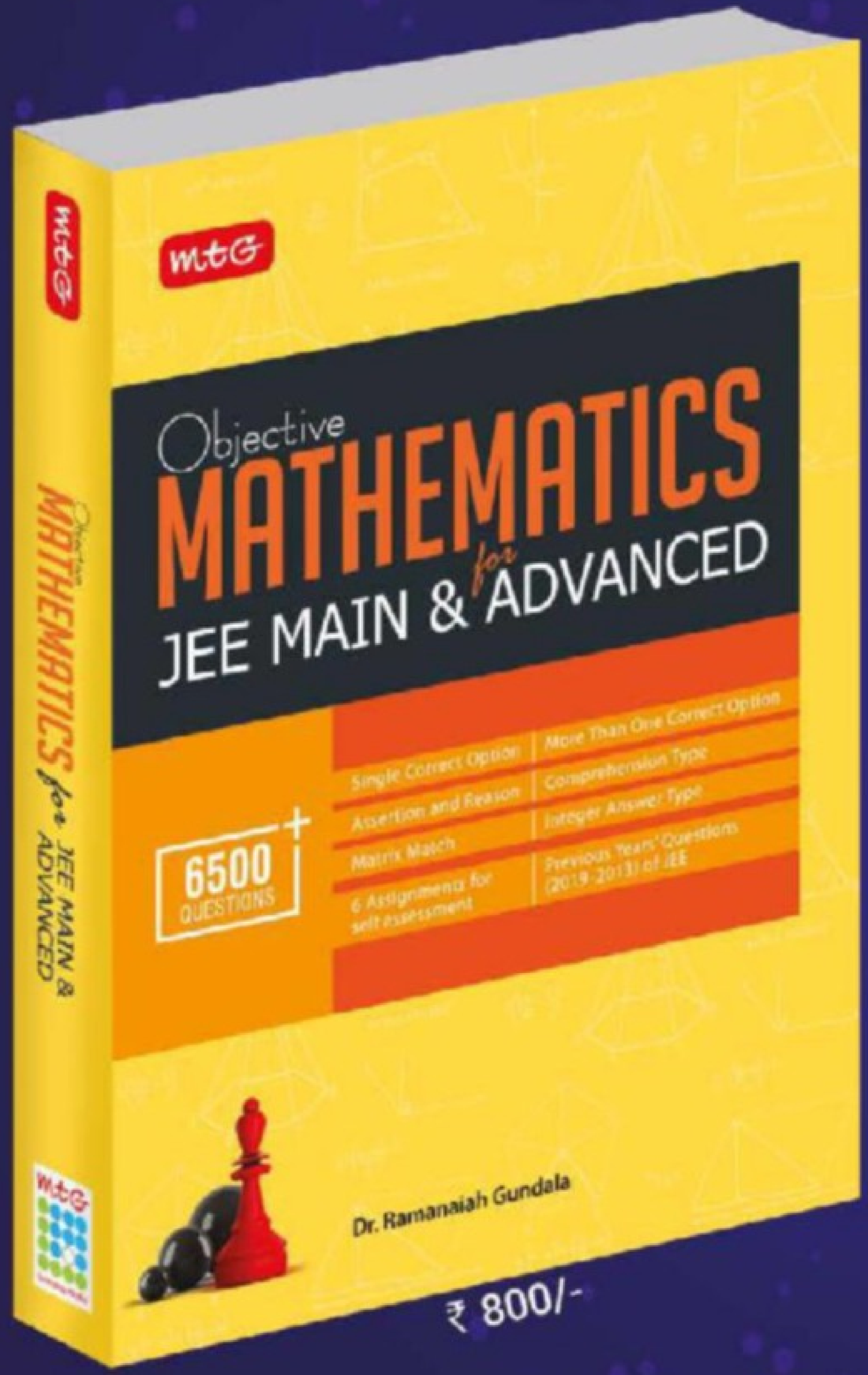
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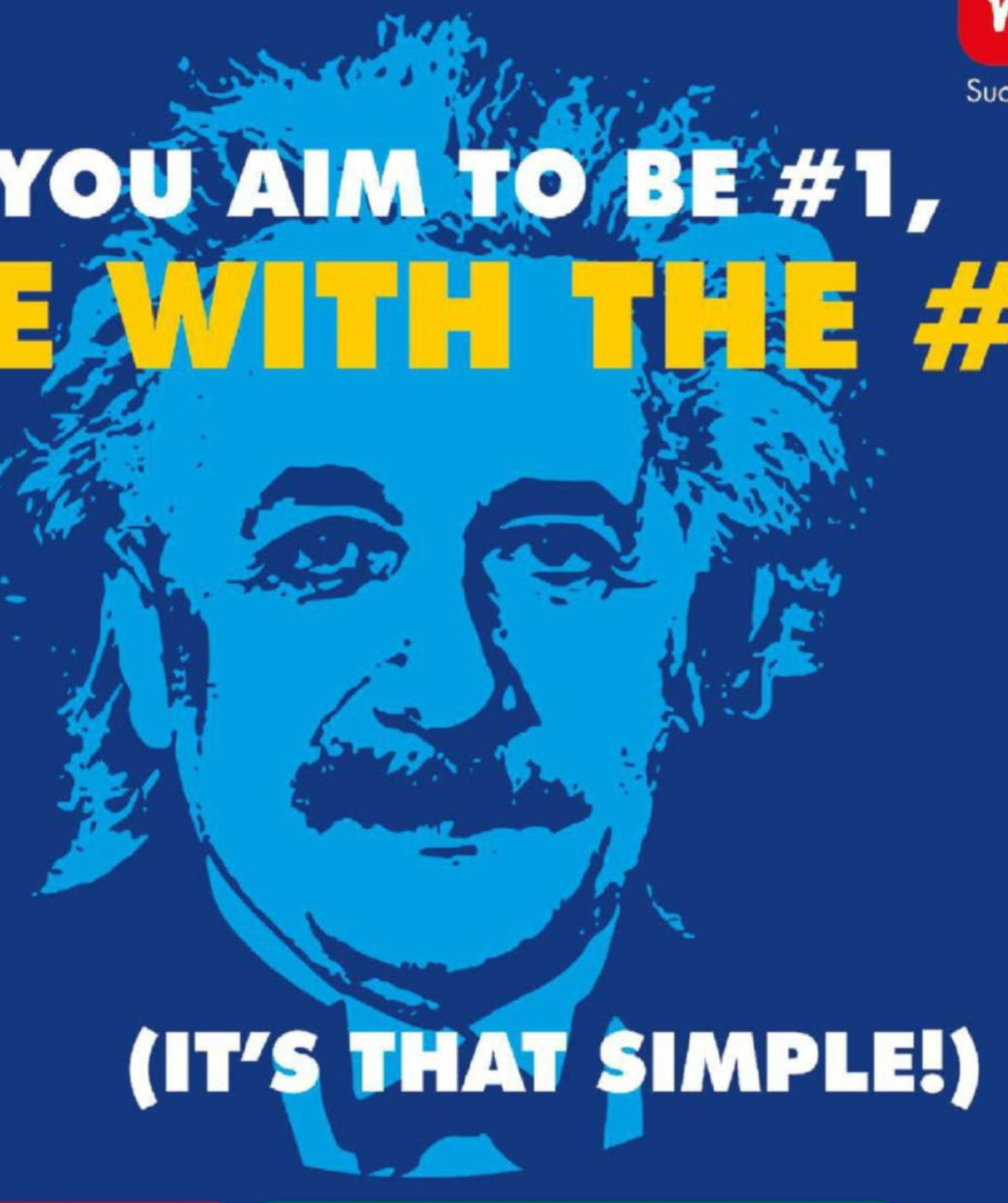

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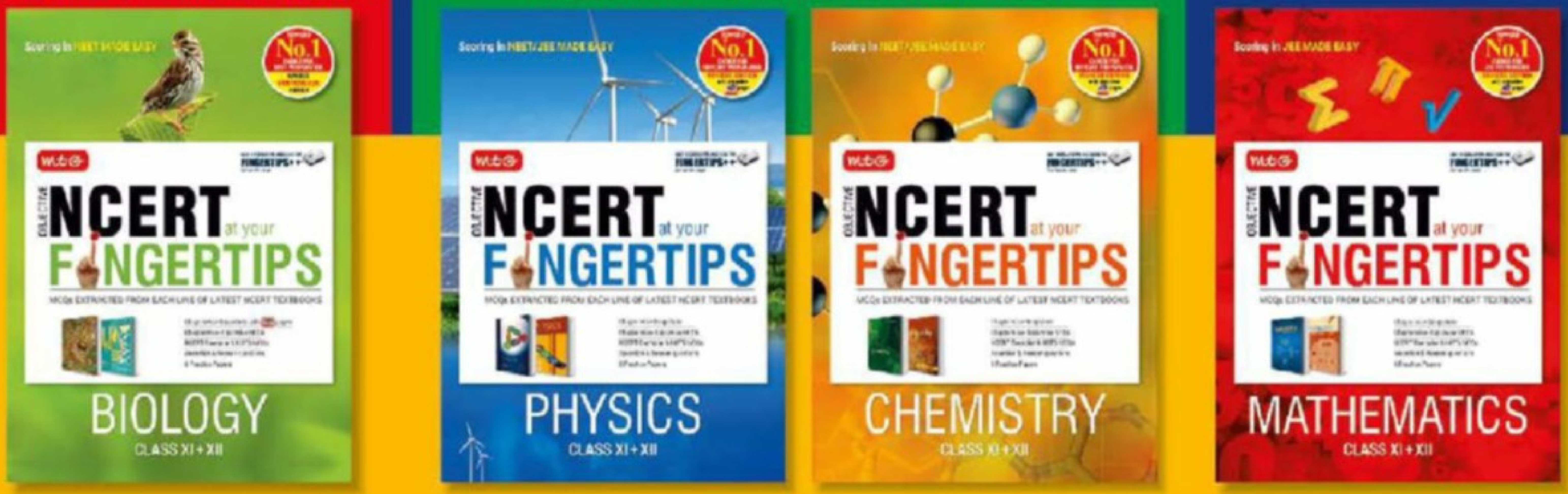
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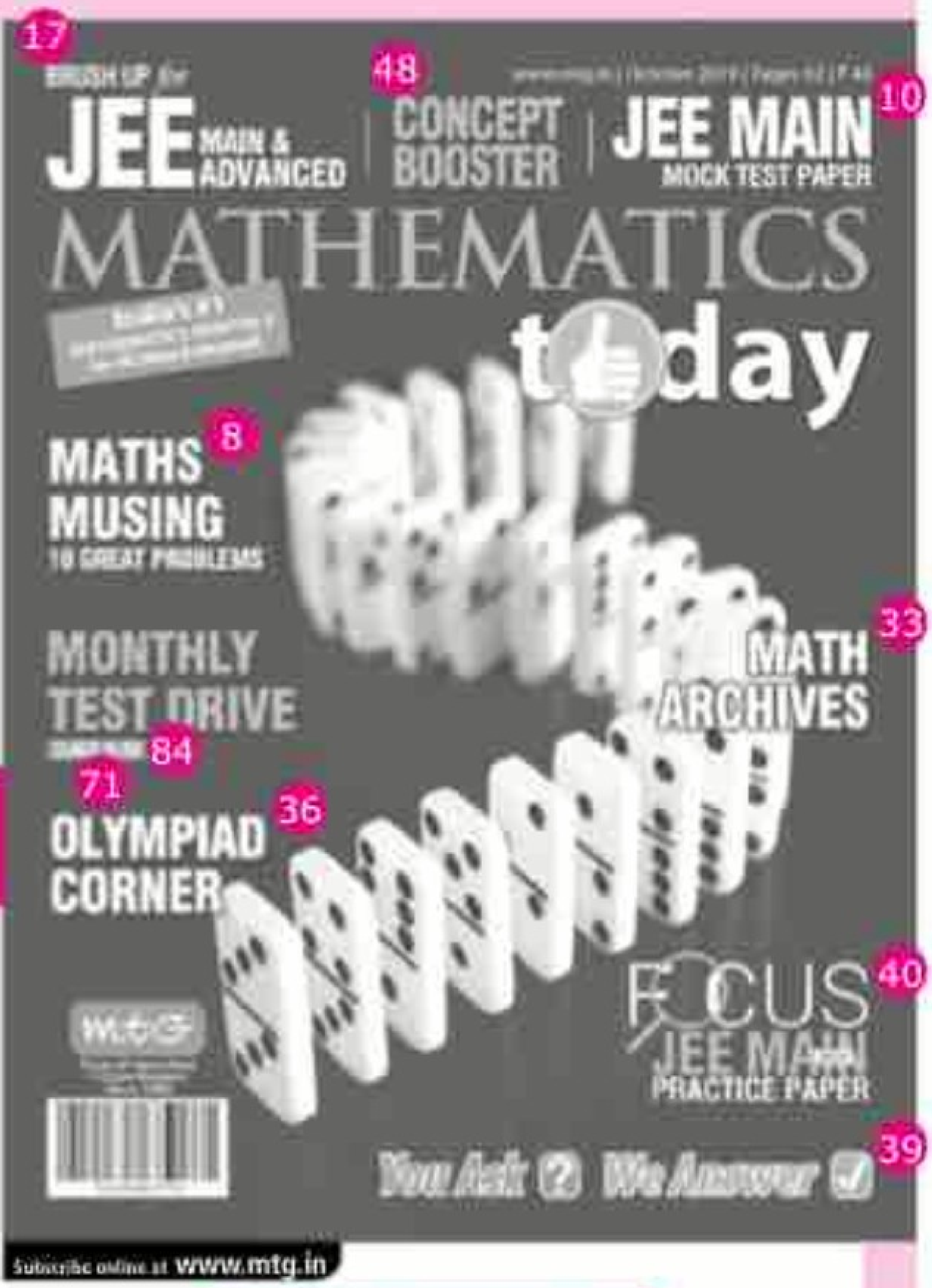
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Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM
Set 202

JEE MAIN

1. If $x^2 - y^2 - 84y = 2012$, $x, y \in N$, then $x - 3y =$
(a) 2 (b) 3 (c) 6 (d) 7

2. Let $S = \{2^0, 2^1, 2^2, ..., 2^{10}\}$. Consider all possible positive differences of elements of S. If M is the sum of all these differences, then the sum of the digits of M is
(a) 24 (b) 27 (c) 30 (d) 31

3. Let S, S_1, S_2 be circles of radii 9, 6, 3 respectively. S_1 and S_2 touch each other externally and S touches both S_1 and S_2 internally. The length of a chord of S which is a common tangent to S_1 and S_2 is
(a) $4\sqrt{14}$ (b) $3\sqrt{14}$ (c) $2\sqrt{14}$ (d) $\sqrt{14}$

4. A fair coin is tossed 12 times. The probability of getting atleast 8 consecutive heads is
(a) $\frac{3}{2^7}$ (b) $\frac{5}{2^7}$ (c) $\frac{5}{2^8}$ (d) $\frac{3}{2^8}$

5. For $x^2 - (a + 3)\,|x| + 4 = 0$ to have real solutions, the range of a is
(a) $(-\infty, -7] \cup (1, \infty)$
(b) $(-3, \infty)$
(c) $(-\infty, -7)$ (d) $[1, \infty)$

JEE ADVANCED

6. A real root of the equation $x^2 - 2^x = 1$ lies in the interval
(a) $(-2, -1)$ (b) $(-1, 2]$
(c) $(2, 3]$ (d) $(3, 4)$

COMPREHENSION

The curve $y = f(x)$ passes through the point $(0, 1)$ and the curve $y = g(x) = \int_{-\infty}^{x} f(t)\,dt$ passes through the point $\left(0, \frac{1}{2}\right)$. The tangents drawn to the curves at the point with equal abscissae intersect on the x-axis.

7. The area bounded by the x-axis, the tangent and normal to the curve $y = f(x)$ at the point where it cuts the y-axis is
(a) $\frac{3}{4}$ (b) 1 (c) $\frac{5}{4}$ (d) $\frac{3}{2}$

8. The area bounded by the curve $y = g(x)$ and the lines $x = 0$, $x = 1$, $y = x$ is
(a) $\frac{e^2 - 1}{2}$ (b) $\frac{e^2 - 2}{3}$
(c) $\frac{e^2 - 3}{3}$ (d) $\frac{e^2 - 3}{4}$

NUMERICAL ANSWER TYPE

9. $A_1 A_2 A_3 \ldots A_{18}$ is a regular 18-gon. B is an external point such that $A_1 A_2 B$ is an equilateral triangle. Then $A_{18} A_1$ and $A_1 B$ are adjacent sides of a regular polygon of n sides. Find the value of n.

MATRIX MATCH

10. Let $S = \{1, 2, 3, ..., 10\}$, then

	Column-I		Column-II
P.	The number of subsets $\{x, y, z\}$ of S so that x, y, z are in A.P., is	1.	15
Q.	The number of subsets $\{x, y, z\}$ of S so that no two of them are consecutive, is	2.	20
R.	The number of subsets $\{x, y\}$ of S so that $x^3 + y^3$ is divisible by 3, is	3.	24
S.	The number of subsets $\{x, y\}$ of S so that $x^2 - y^2$ is divisible by 3, is	4.	56

	P	Q	R	S
(a)	2	4	3	1
(b)	3	2	4	1
(c)	4	1	3	2
(d)	2	4	1	3

◆◆

See Solution Set of Maths Musing 201 on page no. 62

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(for JEE / Olympiad Aspirants)

Myth	Reality
I change book frequently when I am not able to perform better in test, so that I can get good marks.	Changing books is not going to help , try to clear the concept and make your basics strong. **Suggestion**- Always use books recommended by experienced person and do not take many advises.

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Knowledge Quiz - 23

If each permutation of the digits 1, 2, 3, 4, 5, 6 are listed in the increasing order of the magnitude, then find the 289th term.

Please send your detailed solution before 15th October 2019 to **quiz@kcseducate.in** along with your name, father's name, class, school, address and contact number.

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JEE MAIN

with Numerical Value Type Questions

Exam Date : Between 6th to 11th January

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The entire syllabus of Mathematics of JEE-MAIN is being divided into eight units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues. The syllabus for module break-up is given below:

	Topic	Syllabus in Detail
UNIT NO. 5	Differential Calculus	Functions, limits, continuity & differentiability.
	Statistics & Probability	Measures of Dispersion : Calculation of mean, median, mode of grouped and ungrouped data, calculation of standard deviation, variance and mean deviation for grouped and ungrouped data. Probability : Probability of an event, addition and multiplication theorems of probability.
	Coordinate geometry-3D	Coordinate axes and coordinate planes in three dimensions. Coordinate of a point. Distance between two points and section formula.

1. If $af(x+1)+bf\left(\frac{1}{x+1}\right)=x, x\neq -1, a\neq b$, then $f(1)=$

(a) $a+b$ (b) a^2-b^2 (c) $\frac{1}{a+b}$ (d) 0

2. The domain of the definition of the function $f(x)=\sqrt{1-2x}+3\sin^{-1}\left(\frac{3x-1}{2}\right)$, is

(a) [–1/3, 1] (b) (–∞, 1/2]
(c) [–1/3, 1/2] (d) [–1/3, 1/2)

3. Let f be a real valued periodic function defined for all real numbers x such that for some fixed $a > 0$ $f(x+a)=\frac{1}{2}+\sqrt{f(x)-\{f(x)\}^2}$ for all x. Then the period of $f(x)$ is

(a) a (b) $2a$ (c) $3a$ (d) $4a$

4. $\lim_{x\to\infty}\left\{\frac{x^2+1}{x+1}-ax-b\right\}=2$, then

(a) $a = 1, b = 3$ (b) $a = -1, b = 3$
(c) $a = -1, b = -3$ (d) $a = 1, b = -3$

5. If $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$, then $\lim_{h\to 0}\frac{f(1-h)-f(1)}{h^3+3h}=$

(a) $\frac{53}{3}$ (b) $\frac{22}{3}$
(c) 13 (d) None of these

6. If $\lim_{x\to 0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$, then $a+b+c=$

(a) 2 (b) 4 (c) 0 (d) 6

7. $\lim_{x\to\pi/4}\frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}=$

(a) $5\sqrt{2}$ (b) $3\sqrt{2}$
(c) $\sqrt{2}$ (d) None of these

8. The function $f(x)=\begin{cases} x^2/a &, 0\le x<1 \\ a &, 1\le x\le\sqrt{2} \\ \frac{2b^2-4b}{x^2} &, \sqrt{2}\le x<\infty \end{cases}$

is continuous for $0\le x<\infty$, then the most suitable values of a and b are

(a) $a = 1, b = -1$ (b) $a=-1, b=1+\sqrt{2}$
(c) $a = -1, b = 1$ (d) None of these

9. Let $g(x)=\frac{(x-1)^n}{\log\cos^m(x-1)}; 0<x<2,\ m\neq 0, n>0$

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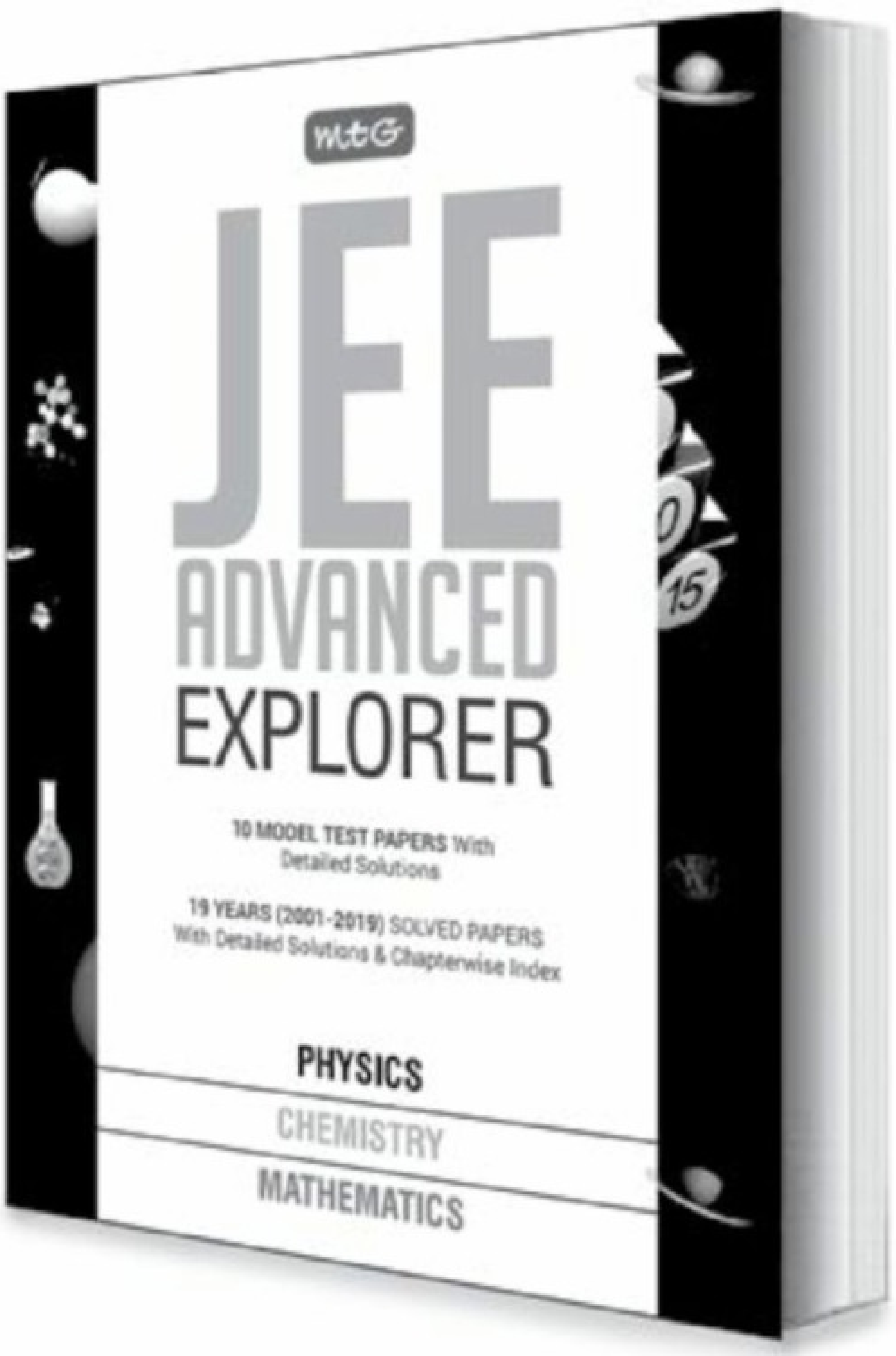
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and let p be the left hand derivative of $f(x) = |x - 1|$ at $x = 1$. If $\lim\limits_{x \to 1^+} g(x) = p,$ then

(a) $n = 1, m = 1$ (b) $n = 1, m = -1$
(c) $n = 2, m = 2$ (d) $n > 2, m = n$

10. Let $f(x) = \cos x$ and $g(x) = [x + 2]$, where [.] denotes the greatest integer function. The value of $(gof)'\left(\frac{\pi}{2}\right)$ is

(a) 1 (b) 0
(c) −1 (d) does not exist

11. If $f(x) = \begin{cases} \dfrac{x(3e^{1/x} + 4)}{2 - e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f(x)$ is

(a) continuous as well as differentiable at $x = 0$
(b) continuous but not differentiable at $x = 0$
(c) differentiable but not continuous at $x = 0$
(d) None of these

12. The age distribution of 400 persons in a colony having median age 32 years is given below :

Age (in years)	Frequency
20-25	110
25-30	x
30-35	75
35-40	55
40-45	y
45-50	30

Then, $x - y$ is

(a) 10 (b) 20 (c) −10 (d) −30

13. If the ratio of mean and median of a certain data is 2 : 3, then the ratio of its mode and mean is

(a) 4 : 3 (b) 7 : 6 (c) 7 : 8 (d) 5 : 2

14. The mean square deviation of a set of n observations $x_1, x_2, ..., x_n$ about a point c is defined as $\frac{1}{n}\sum\limits_{i=1}^{n}(x_i - c)^2.$

The mean square deviation about −2 and 2 are 18 and 10 respectively, then standard deviation of this set of observation is

(a) 3 (b) 2
(c) 1 (d) None of these

15. In a group of 20 males and 5 females, 10 males and 3 females are service holders. The probability that a person selected at random from the group, is a service holder, given that the selected person is a male is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

16. A student answers a multiple choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is p, $0 < p < 1$. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, the probability that he did not tick the answer randomly, is

(a) $\frac{3p}{4p+3}$ (b) $\frac{5p}{3p+2}$ (c) $\frac{5p}{4p+1}$ (d) $\frac{4p}{3p+1}$

17. The point on xy-plane which is equidistant from three points $A(2, 0, 3)$, $B(0, 3, 2)$ and $C(0, 0, 1)$ has the coordinates

(a) (2, 0, 8) (b) (0, 3, 1)
(c) (3, 2, 0) (d) (3, 2, 1)

18. The coordinates of the point where the line segment joining $A(5, 1, 6)$ and $B(3, 4, 1)$ crosses yz-plane are

(a) $\left(0, \frac{17}{2}, \frac{13}{2}\right)$ (b) $\left(0, -\frac{17}{2}, \frac{13}{2}\right)$
(c) $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$ (d) $\left(0, -\frac{17}{2}, -\frac{13}{2}\right)$

19. The point equidistant from the point $O(0, 0, 0)$, $A(a, 0, 0)$, $B(0, b, 0)$ and $(0, 0, c)$ has the coordinates

(a) (a, b, c) (b) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$
(c) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ (d) $\left(\frac{a}{4}, \frac{b}{4}, \frac{c}{4}\right)$

20. $A(3, 2, 0)$, $B(5, 3, 2)$ and $C(-9, 6, -3)$ are the vertices of a triangle ABC. If the bisector of $\angle BAC$ meets BC at D, then coordinates of D are

(a) $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$ (b) $\left(-\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$
(c) $\left(\frac{19}{8}, -\frac{57}{16}, \frac{17}{16}\right)$ (d) None of these

Numerical Value Type

21. The number of integral solutions of the equation $\{x + 1\} + 2x = 4\,[x + 1] - 6$, is

22. If f is an even function defined on the interval $[-5, 5]$, then the total number of real values of x satisfying the equation $f(x) = f\left(\frac{x+1}{x+2}\right)$, is

23. The average marks of boys in a class are 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is

24. If the mean deviation of the numbers 1, 1+ d, 1 + $2d$, ..., 1+ $100d$ from their mean is 255, then d is equal to

25. Two candidates A and B appeared for an interview for two vacancies. The probabilities of their selection are $\frac{1}{4}$ and $\frac{1}{3}$ respectively. The probability that one of them will be selected is

SOLUTIONS

1. (d) : We have, $af(x+1)+bf\left(\frac{1}{x+1}\right)=x$ ....(i)

where $x \in R$, $x \neq -1$

Now, replacing $x+1$ by $\frac{1}{x+1}$ we get,

$$af\left(\frac{1}{x+1}\right)+bf(x+1)=\frac{1}{x+1}-1$$

$$\Rightarrow \quad af\left(\frac{1}{x+1}\right)+bf(x+1)=\frac{-x}{x+1} \qquad \text{...(ii)}$$

Multiplying (i) by a and (ii) by b and subtracting, we get

$$(a^2-b^2)f(x+1)=ax+\frac{bx}{x+1}$$

$$\Rightarrow \quad f(x+1)=\frac{x}{a^2-b^2}\left\{a+\frac{b}{x+1}\right\} \ \forall\, x \in R, x \neq -1$$

$\therefore \quad f(1) = 0$

2. (c) : Here, $f(x)=\sqrt{1-2x}+3\sin^{-1}\left(\frac{3x-1}{2}\right)$

Let $g(x)=\sqrt{1-2x}$ and $h(x)=3\sin^{-1}\left(\frac{3x-1}{2}\right)$

Now, $g(x)$ will be defined when $1 - 2x \geq 0 \Rightarrow 2x \leq 1$

$$\Rightarrow \quad x \leq \frac{1}{2}, \text{ so } D_g=\left(-\infty, \frac{1}{2}\right]$$

Again, $h(x)$ will be defined when $-1 \leq \frac{3x-1}{2} \leq 1$

$$\Rightarrow \quad -2 \leq 3x-1 \leq 2 \Rightarrow -1 \leq 3x \leq 3 \Rightarrow -\frac{1}{3} \leq x \leq 1$$

So, $D_h=\left[-\frac{1}{3}, 1\right]$

Hence, domain of $f(x)$ is $\left(-\infty, \frac{1}{2}\right] \cap \left[-\frac{1}{3}, 1\right]=\left[-\frac{1}{3}, \frac{1}{2}\right]$

3. (b) : Here we have,

$$f(x+a)=\frac{1}{2}+\sqrt{f(x)-\{f(x)\}^2} \qquad \text{...(i)}$$

$$\therefore \quad f(x+a+a)=\frac{1}{2}+\sqrt{f(x+a)-\{f(x+a)\}^2}$$

$$\Rightarrow \quad f(x+2a)=\frac{1}{2}+\sqrt{f(x+a)\{1-f(x+a)\}}$$

$$\Rightarrow \quad f(x+2a)=\frac{1}{2}+\sqrt{\left\{\frac{1}{2}+\sqrt{f(x)-\{f(x)\}^2}\right\}\left\{\frac{1}{2}-\sqrt{f(x)-\{f(x)\}^2}\right\}}$$

[Using (i)]

$$\Rightarrow \quad f(x+2a)=\frac{1}{2}+\sqrt{\frac{1}{4}-\{f(x)-\{f(x)\}^2\}}$$

$$\Rightarrow \quad f(x+2a)=\frac{1}{2}+\sqrt{\left\{f(x)-\frac{1}{2}\right\}^2}$$

$$\Rightarrow \quad f(x+2a)=\frac{1}{2}+\left|f(x)-\frac{1}{2}\right|$$

$$\Rightarrow \quad f(x+2a)=\frac{1}{2}+f(x)-\frac{1}{2}$$

$\Rightarrow \quad f(x + 2a) = f(x)$ for all x

Therefore, $f(x)$ is periodic with period $2a$.

4. (d) : We have, $\lim\limits_{x\to\infty}\left\{\frac{x^2+1}{x+1}-ax-b\right\}=2$

$$\Rightarrow \quad \lim_{x\to\infty}\frac{x^2+1-ax(x+1)-b(x+1)}{x+1}=2$$

$$\Rightarrow \quad \lim_{x\to\infty}\frac{x^2(1-a)-x(a+b)+1-b}{x+1}=2 \qquad \text{...(i)}$$

As the limiting value of the given expression is a finite non-zero number, therefore the degree of both numerator and denominator are same.

$\therefore \quad 1 - a = 0 \ \Rightarrow \ a = 1$

Putting $a = 1$ in (i), we get

$$\lim_{x\to\infty}\frac{-x(1+b)+1-b}{x+1}=2$$

$\Rightarrow \quad -(1 + b) = 2 \ \Rightarrow \ b = -3$

Hence, $a = 1$ and $b = -3$

5. (a) : We have, $\lim\limits_{h\to 0}\frac{f(1-h)-f(1)}{h^3+3h}$

$$=\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}\times\frac{-1}{h^2+3}$$

$$=\lim_{h\to 0}\frac{f(1-h)-f(1)}{(1-h)-1}\times\frac{-1}{h^2+3}=f'(1)\times\left(-\frac{1}{3}\right)=-\frac{1}{3}f'(1)$$

Now, $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$

$\Rightarrow \quad f'(x) = 30x^9 - 56x^7 + 30x^5 - 63x^2 + 6x$

$\Rightarrow \quad f'(1) = 30 - 56 + 30 - 63 + 6 = -53$

$$\text{Hence, } \lim_{h\to 0}\frac{f(1-h)-f(1)}{h^3+3h}=-\frac{1}{3}f'(1)=-\frac{1}{3}\times(-53)=\frac{53}{3}$$

6. (b) : We have, $\lim\limits_{x\to0}\dfrac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$

The above limit exists when $a-b+c=0$ ...(i)

Now, $\lim\limits_{x\to0}\dfrac{ae^x-b\cos x+ce^{-x}}{x\sin x}$ is $\dfrac{0}{0}$ form

$$\therefore\ \lim_{x\to0}\frac{ae^x+b\sin x-ce^{-x}}{\sin x+x\cos x}=2\ \text{[Using L.H. Rule]}$$

When $x\to0$, then the numerator tends to $a-c$.
Thus limit will exist when $a-c=0$ ...(ii)
Now, if $a-c=0$, then

$$\lim_{x\to0}\frac{ae^x+b\sin x-ce^{-x}}{\sin x+x\cos x}\text{ is }\frac{0}{0}\text{ form}$$

$$\therefore\ \lim_{x\to0}\frac{ae^x+b\cos x+ce^{-x}}{2\cos x-x\sin x}=2$$

$$\Rightarrow\ \frac{a+b+c}{2}=2\Rightarrow a+b+c=4$$

7. (a) : We have, $\lim\limits_{x\to\pi/4}\dfrac{4\sqrt2-(\cos x+\sin x)^5}{1-\sin 2x}$

$$=\lim_{x\to\pi/4}\frac{\{(\cos x+\sin x)^2\}^{5/2}-(2)^{5/2}}{(1+\sin 2x)-2}$$

$$=\lim_{x\to\pi/4}\frac{(1+\sin 2x)^{5/2}-(2)^{5/2}}{(1+\sin 2x)-2}$$

$$=\lim_{a\to2}\frac{a^{5/2}-2^{5/2}}{a-2},\ \text{where } a=1+\sin 2x$$

$$=\frac{5}{2}\times2^{5/2-1}=5\sqrt2$$

8. (c) : Given that $f(x)$ is continuous in $0\le x<\infty$. Therefore $f(x)$ is continuous at $x=1$ and $x=\sqrt2$

$$\therefore\ \lim_{x\to1^-}f(x)=\lim_{x\to1^+}f(x)=f(1)$$

$$\text{and }\lim_{x\to\sqrt2^-}f(x)=\lim_{x\to\sqrt2^+}f(x)=f(\sqrt2)$$

$$\Rightarrow\ \lim_{x\to1^-}\frac{x^2}{a}=\lim_{x\to1^+}a=a$$

$$\text{and }\lim_{x\to\sqrt2^-}a=\lim_{x\to\sqrt2^+}\frac{2b^2-4b}{x^2}=\frac{2b^2-4b}{2}$$

$$\Rightarrow\ \frac{1}{a}=a\text{ and }a=\frac{2b^2-4b}{2}$$

$\Rightarrow$ $a=\pm1$ and $b^2-2b=a$

For $a=1,\ b^2-2b=a\Rightarrow b=1\pm\sqrt2$
For $a=-1,\ b^2-2b=a\ \Rightarrow\ b=1$

9. (c) : We have, $f(x)=|x-1|=\begin{cases}x-1, & x\ge1\\ 1-x, & x<1\end{cases}$

Given, p = Left hand derivative of $f(x)$ at $x=1$.

$$\therefore\ p=\lim_{h\to1^-}\frac{f(1+h)-f(1)}{h}=\lim_{h\to1^-}\frac{1-(1+h)-0}{h}$$

$$=\lim_{h\to1^-}\frac{-h}{h}=-1\qquad\text{...(i)}$$

Now, $\lim\limits_{x\to1^+}g(x)=p$ (Given)

$$\Rightarrow\ \lim_{h\to0}g(1+h)=-1\qquad\text{[From (i)]}$$

$$\Rightarrow\ \lim_{h\to0}\frac{h^n}{m\log\cos h}=-1$$

$$\Rightarrow\ \frac{1}{m}\lim_{h\to0}\frac{nh^{n-1}}{-\tan h}=-1\qquad\text{[Using L.H. Rule]}$$

$$\Rightarrow\ \frac{n}{m}\lim_{h\to0}\frac{h^{n-2}}{\left(\dfrac{\tan h}{h}\right)}=1$$

$$\Rightarrow\ n=2\text{ and then }\frac{n}{m}=1\Rightarrow m=n=2$$

10. (d) : We have, $f(x)=\cos x$ and $g(x)=[x+2]$
Now, $(gof)\,x=g(f(x))=[f(x)+2]=[\cos x+2]$
Let $h(x)=(gof)x$

$$\therefore\ Lh'\left(\frac{\pi}{2}\right)=\lim_{k\to0}\frac{h\left(\frac{\pi}{2}-k\right)-h\left(\frac{\pi}{2}\right)}{-k}$$

$$=\lim_{k\to0}\frac{\left[\cos\left(\frac{\pi}{2}-k\right)+2\right]-\left[\cos\frac{\pi}{2}+2\right]}{-k}$$

$$=\lim_{k\to0}\frac{[2+\sin k]-[2]}{-k}=\lim_{k\to0}\frac{2-2}{-k}=0$$

$$Rh'\left(\frac{\pi}{2}\right)=\lim_{k\to0}\frac{h\left(\frac{\pi}{2}+k\right)-h\left(\frac{\pi}{2}\right)}{k}$$

$$=\lim_{k\to0}\frac{\left[\cos\left(\frac{\pi}{2}+k\right)+2\right]-\left[\cos\frac{\pi}{2}+2\right]}{k}=\lim_{k\to0}\frac{1-2}{k}=-\infty$$

So, $h(x)=(gof)x$ is not differentiable at $x=\dfrac{\pi}{2}$.

11. (b) : $f(x)=\begin{cases}\dfrac{x(3e^{1/x}+4)}{2-e^{1/x}}, & x\ne0\\ 0, & x=0\end{cases}$

$$\mathrm{L}f'(0)=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$$

$$=\lim_{h\to 0}\frac{\dfrac{-h\left(3e^{-\frac{1}{h}}+4\right)}{2-e^{-\frac{1}{h}}}-0}{-h}=\frac{0+4}{2-0}=2$$

$$\mathrm{R}f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{h\left(3e^{\frac{1}{h}}+4\right)}{2-e^{\frac{1}{h}}}-0}{h}=\frac{3+0}{0-1}=-3$$

Thus, $\mathrm{L}f'(0)\neq \mathrm{R}f'(0)$.

$\therefore$ $f(x)$ is not differentiable at $x = 0$. But $f(x)$ is continuous at $x = 0$, as both the derivatives are finite.

12. (c) :

Age (in years)	Frequency	Cumulative frequency
20-25	110	110
25-30	x	$110 + x$
30-35	75	$185 + x$
35-40	55	$240 + x$
40-45	y	$240 + x + y$
45-50	30	$270 + x + y$
Total	400	$N = 270 + x + y$

Clearly, $270+x+y=400\Rightarrow x+y=130$ and $\frac{N}{2}=200.$

Given that median is 32, therefore the median class is 30 – 35.

$$\therefore\quad \text{Median } =l_1+\frac{h}{f}\left(\frac{N}{2}-c\right)=30+\frac{5}{75}(200-110-x)$$

$$\Rightarrow\quad 30+\frac{5}{75}(200-110-x)=32\Rightarrow\frac{90-x}{15}=2\Rightarrow x=60$$

Since we have, $x + y = 130$, so $y = 70$

Hence, $x - y = -10$

13. (d) : Given, $\frac{\text{Mean}}{\text{Median}}=\frac{2}{3}$

$\Rightarrow$ 3Mean = 2Median = k(say)

$\Rightarrow$ $\text{Mean}=\frac{k}{3}$ and $\text{Median}=\frac{k}{2}$

Now, Mode = 3 Median-2 Mean $=3\times\frac{k}{2}-2\times\frac{k}{3}=\frac{5}{6}k$

$\therefore$ $\text{Mode : Mean}=\frac{5}{6}k:\frac{k}{3}=5k:2k=5:2$

14. (a) : Given that, the mean square deviation about –2 and 2 are 18 and 10 respectively.

$$\therefore\quad \frac{1}{n}\sum_{i=1}^{n}(x_i+2)^2=18 \text{ and } \frac{1}{n}\sum_{i=1}^{n}(x_i-2)^2=10$$

Now, from $\frac{1}{n}\sum_{i=1}^{n}(x_i+2)^2=18$, we get

$$\frac{1}{n}\sum_{i=1}^{n}x_i^2+4\frac{1}{n}\sum_{i=1}^{n}x_i+4=18\Rightarrow\frac{1}{n}\sum_{i=1}^{n}x_i^2+4\bar{x}=14\quad\text{...(i)}$$

And from, $\frac{1}{n}\sum_{i=1}^{n}(x_i-2)^2=10$, we get

$$\frac{1}{n}\sum_{i=1}^{n}x_i^2-4\frac{1}{n}\sum_{i=1}^{n}x_i+4=10\Rightarrow\frac{1}{n}\sum_{i=1}^{n}x_i^2-4\bar{x}=6\quad\text{...(ii)}$$

Solving (i) and (ii) we get $\frac{1}{n}\sum_{i=1}^{n}x_i^2=10$ and $\bar{x}=1$

$$\therefore\quad \text{S.D.}(\sigma)=\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2-(\bar{x})^2}=\sqrt{10-1}=\sqrt{9}=3$$

15. (a) : Let M be the event that the selected person is a male and $S|M$ is the event that 'the selected person is a service holder when it is known that he is a male. Out of 10 male service holder 1 may be selected in ${}^{10}C_1$ = 10 ways.

So, total number of cases favourable to the event $S|M$ is 10 and total number of exhaustive cases is 20.

Hence, the required probability, $P(S|M)=\frac{10}{20}=\frac{1}{2}$

16. (c) : Let E be the event that the student answer the question correctly and A be the event that he knows the answer.

$\therefore\quad P(E|A)=1;\ P(E|A^c)=\frac{1}{5}$

By the question, $P(A) = p$ $\quad\therefore\quad$ $P(A^c) = 1 - p$

$\therefore$ Required probability = $P(A|E)$

$$=\frac{P(E|A)P(A)}{P(E|A)P(A)+P(E|A^c)P(A^c)}$$

$$=\frac{1\times p}{1\times p+\frac{1}{5}(1-p)}=\frac{p}{\frac{5p+1-p}{5}}=\frac{5p}{4p+1}$$

17. (c) : Let $P(x, y, 0)$ be a point on xy-plane, such that $PA = PB = PC$.

Now, $PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow\quad (x-2)^2+y^2+9=x^2+(y-3)^2+4$

$\Rightarrow\ 4x - 6y = 0 \ \Rightarrow\ 2x - 3y = 0$ ...(i)

Again, $PB = PC \Rightarrow\ PB^2 = PC^2$

$\Rightarrow\ x^2 + (y-3)^2 + 4 = x^2 + y^2 + 1$

$\Rightarrow\ -6y + 12 = 0 \ \Rightarrow y = 2$ ...(ii)

From (i) and (ii), we get, $x = 3$, $y = 2$.

Hence, the required point is (3, 2, 0).

18. (c) : Let yz-plane divides the line segment joining A and B in the ratio $\lambda : 1$. The coordinates of the point C of division are $\left(\frac{3\lambda+5}{\lambda+1}, \frac{4\lambda+1}{\lambda+1}, \frac{\lambda+6}{\lambda+1}\right)$. Since point lies on yz-plane.

$\therefore\ \frac{3\lambda+5}{\lambda+1} = 0 \Rightarrow \lambda = -\frac{5}{3}$

Hence, the coordinates of C are $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$.

19. (b) : Let $P(x, y, z)$ be the required point. Then, $OP = AP = BP = CP$.

Now, $OP = AP \Rightarrow OP^2 = AP^2$

$\Rightarrow\ x^2 + y^2 + z^2 = (x - a)^2 + y^2 + z^2$

$\Rightarrow\ -2ax + a^2 = 0 \ \Rightarrow\ x = \frac{a}{2}$

Similarly, $OP = BP$ and $OP = CP \Rightarrow y = \frac{b}{2}$ and $z = \frac{c}{2}$.

Hence, the required point is $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$.

20. (a) : Let the coordinates of D be (x, y, z).

Now, the length of the side

$$AB = \sqrt{(3-5)^2 + (2-3)^2 + (0-2)^2} = \sqrt{4+1+4} = 3$$

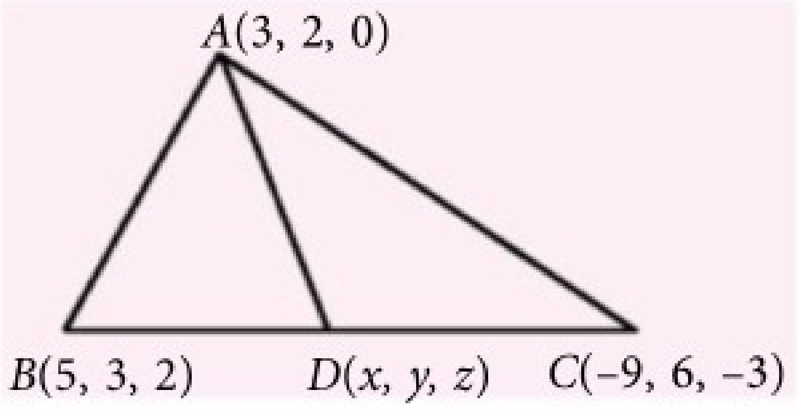
$$AC = \sqrt{(3+9)^2 + (2-6)^2 + (0+3)^2} = \sqrt{144+16+9} = 13$$

Thus, the point D divides the side BC in the ratio 3:13.

$$\therefore\ x = \frac{3(-9)+13(5)}{3+13} = \frac{38}{16},$$

$$y = \frac{3(6)+13(3)}{3+13} = \frac{57}{16}$$

and $z = \frac{3(-3)+13(2)}{3+13} = \frac{17}{16}$



Hence, the coordinates of D are $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$.

21. (1) : We know that $x - [x] = \{x\}$

$\therefore\ \{x + 1\} + 2x = 4[x + 1] - 6$

$\Rightarrow\ x + 1 - [x + 1] + 2x = 4[x + 1] - 6$

$\Rightarrow\ 3x + 1 = 5[x + 1] - 6$

$\Rightarrow\ 3x + 1 = 5[x] + 5 - 6 \ \Rightarrow\ 3x = 5[x] - 2$ ...(i)

$\Rightarrow\ 3([x] + \{x\}) = 5[x] - 2 \Rightarrow\ 3\{x\} = 2[x] - 2$

But, $0 \le \{x\} < 1$

$\therefore\ 0 \le 3\{x\} < 3 \ \Rightarrow\ 0 \le 2[x] - 2 < 3$

$\Rightarrow\ 2 \le 2[x] < 5 \ \Rightarrow\ 1 \le [x] < \frac{5}{2} \ \Rightarrow\ [x] = 1, 2$

Now, from (i), when $[x] = 1$, then $x = 1$ and when $[x] = 2$, then $x = \frac{8}{3}$

22. (4) : Given that $f(x)$ is an even function defined on $[-5, 5]$.

$\therefore\ f(-x) = f(x)\ \forall\ x \in [-5, 5]$

Now, $f(x) = f\left(\frac{x+1}{x+2}\right) \Rightarrow x = \frac{x+1}{x+2}$ or $x = -\frac{x+1}{x+2}$

$\Rightarrow\ x^2 + x - 1 = 0$ or $x^2 + 3x + 1 = 0$

$\Rightarrow\ x = \frac{-1 \pm \sqrt{5}}{2}$ or $x = \frac{-3 \pm \sqrt{5}}{2}$

23. (80) : We are given that the average marks of boys $(\bar{X}_1) = 52$, and that of girls $(\bar{X}_2) = 42$, the combined mean $(\bar{X}) = 50$.

Let n_1 and n_2 be the number of boys and girls respectively in the class.

$$\therefore\ \bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2} \Rightarrow 50 = \frac{52n_1 + 42n_2}{n_1 + n_2} \Rightarrow n_1 = 4n_2$$

Now, percentage of boys $= \frac{n_1}{n_1 + n_2} \times 100 = \frac{4n_2}{5n_2} \times 100 = 80$

24. (10.1) : Let $\bar{x}$ be the mean of the given numbers.

$$\therefore\ \bar{x} = \frac{1 + (1+d) + (1+2d) + ... + (1+100d)}{101}$$

$$\Rightarrow\ \bar{x} = \frac{\frac{101}{2}\{1 + (1+100d)\}}{101} = 1 + 50d$$

$\Rightarrow$ Mean deviation from mean

$$= \frac{1}{101}\sum_{x=0}^{100} \left|1 + xd - (1+50d)\right| = \frac{1}{101}\sum_{x=0}^{100} |x - 50|\, d$$

$$= \frac{d}{101} \times 2\sum_{x=1}^{50} x = \frac{2d}{101} \times \frac{50 \times 51}{2} = \frac{50 \times 51}{101} d$$

Given, $\frac{50 \times 51}{101} d = 255 \Rightarrow d = 10.1$

25. (0.42) : Let E_1 and E_2 be the event of A's and B's selection respectively.

$$\therefore P(E_1) = \frac{1}{4},\ P(E_1^c) = \frac{3}{4},\ P(E_2) = \frac{1}{3},\ P(E_2^c) = \frac{2}{3}$$

The required probability

$$= P[(E_1 \cap E_2^c) \cup (E_2 \cap E_1^c)] = P(E_1 \cap E_2^c) + P(E_2 \cap E_1^c)$$

$$= P(E_1)P(E_2^c) + P(E_2)P(E_1^c) = \frac{1}{4}\cdot\frac{2}{3} + \frac{1}{3}\cdot\frac{3}{4} = \frac{5}{12} = 0.42$$

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Unit 4

Sequences and Series

SEQUENCE

Sequence is a function whose domain is sub-set of N, the set of natural numbers. Sequence gives the images of 1, 2, 3, ..., n as $f_1, f_2, f_3, ...,f_n$ where $f_n = f(n)$. The function $f(n) = (-1)^{n-1}$ gives the sequence whose terms are alternatively 1 and –1. The function $f(n) = n^3$ gives the sequence $1^3, 2^3, 3^3, ..., n^3$ whose domain is 1, 2, 3, ..., n and images $f(1) = 1, f(2) = 2^3, ...,$ so on.

Thus a mapping of $f : N \to C$ defined by $f(n) = t_n$, $n \in N$ is called a sequence to be denoted by

$$\{f(1), f(2), ...\} = \{t_1, t_2, t_3, ... \} = \{t_n\}$$

Thus, 1, 3, 5, 7, ...; 2, 4, 6, 8, ... ; 1, 4, 16, 64, ... ; 27, 9, 3, 1, 1/3, ... are sequences.

Note : Sequence is said to be finite or infinite sequence according as it has finite or infinite number of terms or according as the domain of function is finite subset or an infinite subset of N.

SERIES

By adding or subtracting the terms of a sequence gives a series. Same as sequence, series is said to be finite or infinite series according it has finite or infinite number of terms.

1 + 3 + 5 + 7 + ..., 2 + 4 + 6 + 8 + ...,

$1 - 4 + 16 - 64 + ...$ and $27-9+3-1+\frac{1}{3}$ ..., are series.

The sequence $1^3, 2^3, 3^3, ..., n^3$ gives the series

$1^3 + 2^3 + 3^3 + ... + n^3$ or $1+\frac{1}{2^3}+\frac{1}{3^3}+....$

PROGRESSION

Progression is a sequence whose terms are arranged by a definite rule.

There are three types of progressions (1) Arithmetic progression (A.P.), (2) Geometric progression (G.P.) and (3) Harmonic progression (H.P.).

ARITHMETIC PROGRESSION (A.P.)

A sequence of numbers $< t_n >$ is said to be an arithmetic progression (A.P.) if difference between any two consecutive terms is same. If t_{n-1} and t_n are two consecutive terms of A.P. then $t_n - t_{n-1}$ is a constant $\forall\ n \in N$. This constant is called the common difference of A.P. and is usually denoted by d.

If a is the first term and d is the common difference, then an A.P. can be represented by a, $a + d$, $a + 2d$, $a + 3d$, ...

General Term of A.P.

The general term of an A.P. is denoted by t_n, called n^{th} term and given by $t_n = a + (n - 1)d$.

- The r^{th} term from the end is $(n - r + 1)^{\text{th}}$ term from the beginning.
  $\therefore\ t_{n-r+1} = a + (n - r)d$
- If last term of an A.P. is l, then r^{th} term from the end is $l - (r - 1)d$.

Sum of First *n* Terms of an A.P.

The sum of n terms of the A.P. $a + (a + d) + (a + 2d) + ... + a + (n - 1)d$ is given by

$$S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[a+\overline{a+(n-1)d}] = \frac{n}{2}(a+l),$$

By : **R. K. Tyagi**, Retd. Principal, HOD Maths, Samarth Shiksha Samiti, New Delhi where l is the last term of the A.P. and given by $l = a + (n - 1)d = t_n$

Arithmetic Mean (A.M.)

- If a and b are two real numbers then their arithmetic mean *i.e.*, A.M. is given by $\frac{a+b}{2}$.
- If a, x, b are in A.P. then $x = \frac{a+b}{2}$ is the A.M. of a and b.
- If $a, x_1, x_2, x_3, ..., x_n, b$ are in A.P. then $x_1, x_2, ..., x_n$ are n A.M.'s between a and b, then $d = \frac{b-a}{n+1}$.

$$\therefore \quad x_1 = a + d = \frac{na+b}{n+1}, \quad x_2 = a + 2d = \frac{(n-1)a+2b}{n+1},$$

$$x_3 = a + 3d = \frac{(n-2)a+3b}{n+1}, \quad .....$$

$$....., \quad x_n = b - d = \frac{a+nb}{n+1} \quad \text{or} \quad a + \frac{n(b-a)}{n+1} = a + nd$$

Properties of A.P.

- Three numbers a, b, c are in A.P. if and only if $2b = a + c$.
- If the number of terms of an A.P. is even then A.M. of two most middle terms is A.M. of first and last terms. *i.e.*, if $a_1, a_2, a_3, ..., a_{n-1}, a_n, a_{n+1}, a_{n+2}, ..., a_{2n-1}, a_{2n}$ are in A.P. then $a_1 + a_{2n} = a_n + a_{n+1}$ Moreover, $a_1 + a_{2n} = a_2 + a_{2n-1} = ... = a_n + a_{n+1}$ *i.e.*, sum of the first and last term is equal to the sum of the terms equidistant from beginning and end.
- If the number of terms of an A.P. is odd then sum of its first and last term is twice the middle term. *i.e.*, if $a_1, a_2, ..., a_{n-1}, a_n, a_{n+1}, a_{n+2}, ..., a_{2n+1}$ are in A.P., then $a_1 + a_{2n+1} = 2 \cdot a_{n+1}$ moreover, $a_1 + a_{2n+1} = a_2 + a_{2n} = ...$
  $= a_n + a_{n+2} = 2a_{n+1}$
  Also, $a_1 + a_2 + a_3 + ... + a_{2n+1}$
  $=$ Number of terms $\times\, a_{n+1}$
  i.e., $S_{2n+1} = (2n + 1)a_{n+1}$
  e.g., 1, 3, 5, 7, 9, 11, 13 are in A.P. then
  $S_7 = 7 \times a_4 = 7 \times 7 = 49$.
- If the terms of an A.P. are chosen at regular intervals then the new sequence so formed also forms an A.P.
- If $a_1, a_2, ..., a_n$ and $b_1, b_2, ..., b_n$ are two A.P.'s then $a_1 \pm b_1, a_2 \pm b_2, ..., a_n \pm b_n$ are also in A.P. with common difference $d_1 \pm d_2$, where d_1 and d_2 respectively are the common difference of the given A.P.'s.
- If $S_1, S_2, S_3, ..., S_n$ denote the sum of first term, sum of first two terms, sum of first three terms, ..., the sum of n terms respectively of an A.P., then
  (a) $t_1 = S_1$
  (b) common difference, $d = S_2 - 2S_1$
  (c) $t_n = S_n - S_{n-1}$

Invariance of A.P.

If a, b, c are in A.P. with common difference d then c, b, a ; $a + \alpha, b + \alpha, c + \alpha$; $a - \alpha, b - \alpha, c - \alpha$; $\alpha a, \alpha b, \alpha c$ and $\frac{a}{\alpha}, \frac{b}{\alpha}, \frac{c}{\alpha}$ are also in A.P. with common difference$-d$, d, d, αd, $\frac{d}{\alpha}$ respectively, where α is a fixed non-zero real number.

- For odd number of terms in A.P., select (take) the terms as
  .... $a - 2d, a - d, a, a + d, a + 2d$, ...
  (common difference d)
- For even number of terms in A.P., take the terms as
  ..., $a - 3d, a - d, a + d, a + 3d$, ...
  (common difference $2d$)

Special Series or Identities in Natural Numbers

- $1+2+3+....+n = \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$
- $1^2+2^2+3^2+...+n^2 = \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$
- $1^3+2^3+3^3+...+n^3 = \sum_{r=1}^{n} r^3 = \left(\frac{n(n+1)}{2}\right)^2$
- $1+3+5+....+(2n-1) = \sum_{r=1}^{n}(2r-1) = n^2, t_r = 2r-1$
- $1^2+3^2+5^2+....$ to n terms $= \frac{n(4n^2-1)}{3}$

$$\left[\because t_n = (2n-1)^2 = 4n^2 - 4n + 1 \Rightarrow \Sigma t_n = \frac{n(4n^2-1)}{3}\right]$$

General Term for Some Special Series or Sequences and their Sums

$1n, 2(n - 1), 3(n - 2), ..., (n - 1)2, n(1)$

$\therefore \quad t_r = r(n - \overline{r-1})$

$$\therefore \quad S_n = \sum_{r=1}^{n} r(n-r+1) = (n+1)\sum_{r=1}^{n} r - \sum_{r=1}^{n} r^2$$

$$= \frac{(n+1)n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)(n+2)}{6}$$

GEOMETRIC PROGRESSION (G.P.)

A sequence in which the ratio of any two terms is same *i.e.*, if the ratio of its each term to its preceding term is always a fixed quantity (number) is called G.P. The fixed quantity is called common ratio denoted by r. If a be the first term then $a, ar, ar^2, ... ar^{n-1}$ is a sequence of G.P.

General Term of G.P.

- The general term of any sequence is usually denoted by t_n, which is the n^{th} term.
  $\therefore$ n^{th} term of G.P., $a, ar, ar^2, ...$ is
  $$t_n = ar^{n-1}, \text{ where } r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = ... = \frac{t_n}{t_{n-1}}$$
- If a G.P. having n terms, then R^{th} term from the end $= (n - R + 1)^{\text{th}}$ term from the starting (beginning) $= ar^{n-R}$ and the R^{th} term from the end of a G.P. whose last term is l and common ratio r is $l \cdot \left(\frac{1}{r}\right)^{R-1}$.

Sum of First *n* Terms of a G.P.

If a be the first term and r be the common ratio then sum of the terms $a, ar, ar^2, ..., ar^{n-1}$ is given by

$S_n = \frac{a(1-r^n)}{1-r} = \frac{a-lr}{1-r}$, where l is the last term and $|r| < 1$ *i.e.*, $-1 < r < 1$

and $S_n = \frac{a(r^n-1)}{r-1} = \frac{lr-a}{r-1}$, where l is the last term and $|r| > 1$, *i.e.*, $r < -1$ or $r > 1$

If $r = 1$, then $a, ar, ar^2, ..., ar^{n-1}$ reduces to $a, a, a, ..., a$ (n factors) then $S_n = n \cdot a$.

Sum of Infinite Terms of G.P.

- $S_\infty = \frac{a}{1-r}$ when $|r| < 1$ *i.e.*, $-1 < r < 1$
- S_∞ does not exist if r is either 1 or greater than 1.

Geometric Mean (G.M.)

- If a, x, b are in G.P. then x is called G.M. between a and b and given by $x = \sqrt{ab}, a > 0, b > 0$.
- If $a, x_1, x_2, ..., x_n, b$ are in G.P. then $x_1, x_2, x_3, ..., x_n$ are the n geometric means between a and b and if r is common ratio then terms $x_1, x_2, x_3, ..., x_n$ reduces to $ar, ar^2, ..., ar^n$ where $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ then
  $$x_1 = ar = \text{first G.M.} = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$
  $$x_2 = ar^2 = \text{second G.M.} = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$
  ....
  $$x_n = ar^n \text{ or } \frac{b}{r} = b\left(\frac{a}{b}\right)^{\frac{1}{n+1}} \text{ or } a \cdot \left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

Properties of G.P.

- If we take reciprocal of the terms $a, ar, ar^2, ..., ar^{n-1}, ...$ we get the new sequence $\frac{1}{a}, \frac{1}{ar}, \frac{1}{ar^2}, ..., \frac{1}{ar^{n-1}}, ...$ then the sequence remains in G.P. with common ratio $1/r$.
- In a finite G.P. the product of term equidistant from the starting (beginning) and the end is always the same and is equal to the product of first and last term *i.e.*, if terms of G.P. are $a_1, a_2, ..., a_n$ then $a_1 a_n = a_2 a_{n-1}, = a_3 a_{n-2} = ... = a_R\, a_{n-R+1}$.
- If $a_1, a_2, a_3, ..., a_n$ are in G.P. $(a_i > 0, i = 1, 2, 3, ..., n$ and $a_i \neq 0)$, then $\log a_i$ *i.e.*, $\log a_1, \log a_2, \log a_3, ..., \log a_n$ ... are in A.P. and vice versa.
- If the first term of a G.P. of n terms is a with last term l, then product of all the terms of G.P. is $(al)^{n/2}$
  i.e., $a, ar, ar^2, ..., ar^{n-1} = a^n r^{\frac{n(n-1)}{2}} = (a^2)^{\frac{n}{2}} (r^{n-1})^{\frac{n}{2}}$
  $= (a^2 r^{n-1})^{n/2} = (a \cdot ar^{n-1})^{n/2} = (al)^{n/2}$
- If a_i and b_i $(i = 1, 2, ..., n)$ are two G.P.'s with common ratio r_1 and r_2 respectively, then
  (a) $a_1 b_1, a_2 b_2, a_3 b_3, ..., a_n b_n$ is a G.P. whose common ratio is $r_1 r_2$ $(\neq 0)$
  (b) $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, ..., \frac{a_n}{b_n}$ is a G.P. whose common ratio is $\frac{r_1}{r_2} (r_1, r_2 \neq 0)$
- If a, b, c are in G.P. then square of middle term = product of extreme terms.
  i.e., $b^2 = ac$
- If p^{th}, q^{th} and r^{th} terms of a G.P. are in G.P. then p, q, r are in A.P.
- If a, b, c are in A.P. and G.P. both, then each term is identical *i.e.*, $a = b = c$.
- If a, b, c are in A.P. then $\alpha^a, \alpha^b, \alpha^c$ $(\alpha \neq \pm 1)$ will be in G.P.
- Let the first term of a G.P. is positive if $r > 1$, then it is increasing G.P. but if $0 < r < 1$ then it is decreasing G.P.
- Let the first term of a G.P. is negative, if $r > 1$, then it is a decreasing G.P. but for $0 < r < 1$ it is increasing G.P.

Invariance of G.P.

- If a, b, c are in G.P. with common ratio r then c, b, a ; $\alpha a, \alpha b, \alpha c$; $\frac{a}{\alpha}, \frac{b}{\alpha}, \frac{c}{\alpha}$ $(\alpha \neq 0)$ are in G.P. with common ratio $\frac{1}{r}, r, r$ respectively.
- If a, b, c are in G.P. with common ratio r then a^m, b^m, c^m, $m \in N$ also in G.P. with common ratio r^m.

HARMONIC PROGRESSION (H.P.)

A sequence is an H.P. if the reciprocal of its terms form an A.P. *e.g.*, $\frac{3}{2}, 1, \frac{3}{4}$ are in H.P. since $\frac{2}{3}, 1, \frac{4}{3}$ are in A.P.

General Term of H.P.

As we know that $a, a+d, a+2d, ..., a+(n-1)d, ...$ are terms of A.P.

$\therefore$ Terms of H.P. be $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, ..., \frac{1}{a+(n-1)d}$

$\therefore$ General term t_n of H.P. is $\frac{1}{a+(n-1)d}$.

Harmonic Mean (H.M.)

- If a, x, b are in H.P., then $x = \frac{2ab}{a+b}$ is called the harmonic mean between a and b.
- The harmonic mean of n non zero numbers $a_1, a_2, a_3, ..., a_n$ is given by $\frac{1}{H} = \frac{\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+...+\frac{1}{a_n}}{n}$
- If a and b be two given numbers and set of n numbers *i.e.*, harmonic means $H_1, H_2, H_3, ..., H_n$ be inserted between these two numbers then $a, H_1, H_2, H_3, ..., H_n, b$ are in H.P.

  $\Rightarrow \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, ..., \frac{1}{H_n}, \frac{1}{b}$ are in A.P.

  This A.P. has $(n+2)$ terms. Let D be the common difference, then $(n+2)^{\text{th}}$ term is given by

  $$t_{n+2} = \frac{1}{b} = \frac{1}{a} + (n+1)D \Rightarrow D = \frac{a-b}{(n+1)ab}$$

  $$\therefore H_1 = \frac{(n+1)ab}{a+nb}, H_2 = \frac{(n+1)ab}{2a+(n-1)b}, ...$$

  $$H_n = \frac{(n+1)ab}{na+b}$$

  or $$\frac{1}{H_n} = \frac{1}{a} + nD = \frac{1}{a} + \frac{n(a-b)}{(n+1)ab} = \frac{na+b}{(n+1)ab}$$

Properties of H.P.

- No term of H.P. can be zero and there is no general formula for calculating the sum of n terms of H.P.
- If H is the H.M. between two numbers a and b then
  (a) $(H-2a)(H-2b) = H^2$
  (b) $\frac{H+a}{H-a} + \frac{H+b}{H-b} = 2$
- If $x_1, x_2, x_3, ..., x_n$ are in H.P., then $x_1x_2 + x_2x_3 + x_3x_4 + ... + x_{n-1}x_n = (n-1)\, x_1 x_n$.

Invariance of H.P.

If a, b, c are in H.P., then c, b, a; $\alpha a, \alpha b, \alpha c$; $\frac{a}{\alpha}, \frac{b}{\alpha}, \frac{c}{\alpha}$ $(\alpha \neq 0)$ are also in harmonic progression.

Formation of Progression

- Two consecutive terms determine the required progression.

  $a, b \Rightarrow a, b, 2b-a \in$ A.P.

  $a, b \Rightarrow a, b, \frac{b^2}{a} \in$ G.P.

  $a, b \Rightarrow a, b, \frac{ab}{2a-b} \in$ H.P.
- If a, b, c are in A.P. and b, c, a are in G.P. then c, a, b are in H.P.

Relationship between A.M., G.M. and H.M.

a, A.M., $b \in$ A.P., a, G.M., $b \in$ G.P. and a, H.M., $b \in$ H.P.

Let A.M. $= A$, G.M. $= G$ and H.M. $= H$

Now, $a, A, b \in$ A.P. $\Rightarrow A = \frac{a+b}{2}$

Similarly, $G = \sqrt{ab}$ and $H = \frac{2ab}{a+b}$

$$\therefore A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{(\sqrt{a}-\sqrt{b})^2}{2} \geq 0$$

$\Rightarrow A \geq G$ ... (i)

$$\text{And } G - H = \sqrt{ab} - \frac{2ab}{a+b} = \sqrt{ab}\left(\frac{a+b-2\sqrt{ab}}{a+b}\right)$$

$$= \frac{\sqrt{ab}(\sqrt{a}-\sqrt{b})^2}{a+b} \geq 0$$

$\Rightarrow G \geq H$ ... (ii)

Thus, from (i) and (ii), we have $A \geq G \geq H$, equality holds only when $a = b$.

- A, G, H form a G.P. $\Rightarrow G^2 = AH$.
- The equation having a, b as its roots is $x^2 - (a+b)x + ab = 0$

$$\Rightarrow x^2 - 2Ax + G^2 = 0 \left(\text{using } A = \frac{a+b}{2}, G = \sqrt{ab}\right)$$

$$\Rightarrow \quad \text{Roots } a, b \text{ are } \frac{2A \pm \sqrt{(4A^2 - 4G^2)}}{2}$$

i.e., $A + \sqrt{A^2 - G^2},\ A - \sqrt{A^2 - G^2}$

- If A, G, H are A.M., G.M. and H.M. between the three numbers a, b, c respectively then equation whose roots are a, b, c is given by
$x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc = 0$
$$\Rightarrow \quad x^3 - 3Ax^2 + \frac{3G^3}{H}x - G^3 = 0$$
$$\left\{\begin{array}{l} \text{where } A = \dfrac{a+b+c}{3},\ G = (abc)^{1/3} \\ \text{and } \dfrac{1}{H} = \dfrac{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}{3} \end{array}\right\}$$
- If A, G, H be A.M., G.M., H.M. between the numbers a and b then
$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \left\{\begin{array}{l} A = \dfrac{a+b}{2}, \text{ when } n = 0 \\ G = \sqrt{ab}, \text{ when } n = -\dfrac{1}{2} \\ H = \dfrac{2ab}{a+b} \quad \text{when } n = -1 \end{array}\right\}$$
- If two A.M.'s A_1 and A_2, two G.M.'s G_1 and G_2 and two H.M.'s H_1 and H_2 are inserted between any two numbers (say a and b) then
$$\frac{G_1G_2}{H_1H_2} = \frac{A_1 + A_2}{H_1 + H_2}$$
- If the number of terms of any A.P./G.P./H.P. is odd, then A.M./G.M./H.M of first and last term is middle term of the series respectively.
- If the number of terms of any A.P./G.P./H.P. is even, then A.M./G.M./H.M. of middle two terms is A.M./G.M./H.M. of the first and last terms respectively.

ARITHMETICO-GEOMETRIC PROGRESSION (A.G.P.)

The collection of A.P. and G.P. is said to be A.G.P.

If $a_1, a_2, a_3, ..., a_n$ is an A.P. with common difference d and $b_1, b_2, b_3, ..., b_n$ is a G.P. with common ratio r, then the sequence $a_1b_1, a_2b_2, a_3b_3, ..., a_nb_n$ is said to be an arithmetico-geometric sequence.

Now $a_1, a_2, a_3, ..., a_n$ becomes $a_1 = a$(say), $(a + d)$, $(a + 2d)$, ... $a + (n - 1)d$ and $b_1, b_2, b_3, ..., b_n$ becomes $b_1 = b$(say), $br, br^2, ..., br^{n-1}$ then $a_1b_1, a_2b_2, a_3b_3, ..., a_nb_n$ becomes $ab, (a + d)br, (a + 2d)br^2, ..., (a + (n - 1)d)br^{n-1}$ and $a, (a + d)r, (a + 2d)r^2, ..., [a + (n - 1)d]r^{n-1}$ for $b = 1$.

General Term of A.G.P.

As the terms of A.G.P. are $a, (a + d)r, (a + 2d)r^2, ...$

$\therefore \quad t_n = [a + (n - 1)d]r^{n-1}$.

Sum of A.G.P.

The sum of n terms of an A.G.P. with terms $a, (a + d)r, (a + 2d)r^2$ is given by

- $$S_n = \left\{\frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{\{a+(n-1)d\}r^n}{1-r}, r \neq 1\right\}$$
- Sum to infinite terms of a A.G.P. when $|r| < 1$ *i.e.*, $-1 < r < 1$ then $r^n \to 0$.
$$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} = \frac{a + (d-a)r}{(1-r)^2}$$

Properties of A.G.P.

A.G.P. is given by $a, (a + d)r, (a + 2d)r^2, (a + 3d)r^3, ...$

- If $r = 1$, then A.G.P. reduces to A.P. with terms $a, a + d, a + 2d, ...$
- If $d = 0$, then A.G.P. reduces to G.P. with terms $a, ar, ar^2, ...$

Method to Find the General Term (t_n) of G.P. and A.G.P. by Using Successive Order of Difference

If few terms of a series are given and if p^{th} order of differences of these terms form a G.P., with common ratio r, then general form of the two series is taken as
$$t_n = ar^{n-1} + f(n),$$
where $f(n)$ is a rational integral function of n of $(p - 1)$ dimensions.

$\therefore$ Consider the general term as follows (given):

$t_n = ar^{n-1}$ for direct G.P.

$t_n = ar^{n-1} + b$ if first order successive difference is in G.P.

$t_n = ar^{n-1} + bn + c$ if second order successive difference is in G.P.

$t_n = ar^{n-1} + bn^2 + cn + d$

if third order successive difference is in G.P., where $a, b, c, d, ...$ are constants which can be obtained by using $t_1, t_2, t_3, ...$ by putting $n = 1, 2, 3, ...$ respectively in above relations.

V_n Method

The method is used to find the sum of the series of the forms

(A) $a_1a_2 ... a_r + a_2a_3 ... a_{r+1} + ... + a_na_{n+1} ... a_{n+r-1}$

(B) $$\frac{1}{a_1a_2 ... a_r} + \frac{1}{a_2a_3 ... a_{r+1}} + ... + \frac{1}{a_n a_{n+1} ... a_{n+r-1}}$$

where $a_1, a_2, ..., a_n \in$ A.P.

In order to find the sum of terms of the series (A) we use the following steps.

Step-I : Let the common difference of A.P. = d.

$\therefore \quad a_n = a_1 + (n - 1)d$

Let S_n denotes the sum of n terms and $t_n = n^{\text{th}}$ term of the series. Define, $t_n = a_n\, a_{n+1} \ldots a_{n+r-1}$.

Step-II : Define V_n from t_n by taking next term of the series (*i.e.* one extra term in t_n for V_n)

i.e., $V_n = t_n \cdot a_{n+r} = a_n\, a_{n+1} \ldots\ldots a_{n+r-1}\, a_{n+r}$

$\therefore \quad V_{n-1} = a_{n-1}\, a_n \ldots\ldots a_{n+r-2}\, a_{n+r-1}$

(replacing n by $(n - 1)$ in V_n)

Step-III : Make the difference between V_n and V_{n-1} to develop the relationship between t_n, V_n and V_{n-1}.

$$\therefore \quad V_n - V_{n-1} = (a_n\, a_{n+1}\, a_{n+2} \ldots a_{n+r-1})(a_{n+r} - a_{n-1})$$

$$\Rightarrow \quad V_n - V_{n-1} = t_n[a_1 + (n + r - 1)d - a_1 - (n - 2)d]$$

$$\Rightarrow \quad V_n - V_{n-1} = (r + 1)d \cdot t_n$$

$$\Rightarrow \quad t_n = \frac{1}{(r+1)d}[V_n - V_{n-1}]$$

Now put n = 1, 2, ..., n to get t_1, t_2, ..., t_n and add them columnwise to obtain

$$S_n = t_1 + t_2 + \ldots + t_n = \frac{1}{(r+1)d}[V_n - V_0]$$

$$\Rightarrow \quad S_n = \frac{1}{(r+1)(a_2 - a_1)}(a_n\, a_{n+1} \ldots a_{n+r} - a_0 a_1 a_2 \ldots a_r)$$

where $a_0 = a_1 - d$, which is required sum of n terms of the series of form (A).

Also for series (B), we have

$$t_n = \frac{1}{a_n\, a_{n+1} \ldots\ldots a_{n+r-2} a_{n+r-1}}$$

$$\therefore \quad V_n = \frac{1}{a_{n+1}\, a_{n+2} \ldots\ldots a_{n+r-2}\, a_{n+r-1}}$$

(Avoiding first term from t_n)

$$\text{and } V_{n-1} = \frac{1}{a_n a_{n+1} \ldots\ldots a_{n+r-3} a_{n+r-2}}$$

(Replacing n by $(n - 1)$ in V_n)

$$\therefore \quad V_n - V_{n-1} = \frac{a_n - a_{n+r-1}}{a_n\, a_{n+1} \ldots a_{n+r-1}} = t_n(1-r)\, d$$

$$\Rightarrow \quad t_n = \frac{1}{d(r-1)}[V_{n-1} - V_n]$$

Now, put n = 1, 2, ..., n and add columnwise to get

$$S_n = t_1 + t_2 + \ldots\ldots + t_n = \frac{1}{(r-1)d}(V_0 - V_n),$$

where V_0 is obtained by putting $n = 1$ in V_{n-1}.

Method to Find $\sum_{r=1}^{n} \frac{1}{t_r}$ when $\sum_{r=1}^{n} t_r$ is Known

We discuss the method with the help of an example.

$$\text{Let } \sum_{r=1}^{n} t_r = \frac{n(n+1)(n+2)}{3} = S_n$$

$$\therefore \quad t_n = S_n - S_{n-1} = n(n+1)$$

$$\therefore \quad \frac{1}{t_n} = \frac{1}{n(n+1)}$$

$$\text{Let } V_n = \frac{1}{n+1} \quad \text{(Avoiding first term of } t_n) \qquad \ldots\text{(i)}$$

$$\therefore \quad V_{n-1} = \frac{1}{n} \quad \text{(Replacing } n \text{ by } (n - 1) \text{ in } V_n) \qquad \ldots\text{(ii)}$$

Subtracting (ii) from (i), we get

$$V_n - V_{n-1} = -\frac{1}{n(n+1)} = -\frac{1}{t_n} \Rightarrow \frac{1}{t_n} = V_{n-1} - V_n \quad \ldots(*)$$

Now putting n = 1, 2, 3, ..., n in (*) and adding columnwise we get

$$\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \ldots + \frac{1}{t_n} = (V_0 - V_n) = 1 - \frac{1}{n+1} = \frac{n}{n+1}$$

$$\therefore \quad \sum_{r=1}^{n} \frac{1}{t_r} = \frac{n}{n+1}$$

$$\text{and } \lim_{n\to\infty} \sum \frac{1}{t_r} = \lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = 1$$

To Find the Coefficients of Various Exponents of the Variable in the Product of Linear Factors

$(x + 1)(x + 2)(x + 3) \ldots (x + n)$ (n linear factors)

$= x^n$ + (sum of constant numbers)x^{n-1}

+ (sum of the product of numbers taken 2 at a time)x^{n-2}

+ (sum of the product of numbers taken 3 at a time)x^{n-3}

+ ....

+ (sum of the product of numbers taken $(n - 1)$ at a time)x^1

+ product of the numbers taken all at a time

$$= x^n + (1 + 2 + 3 + \ldots + n)x^{n-1}$$

$$+ \frac{1}{2}[(1 + 2 + 3 + \ldots + n)^2 - (1^2 + 2^2 + \ldots + n^2)]\, x^{n-2}$$

$$+ \frac{(n-2)(n-1)\, n^2 (n+1)^2}{48} x^{n-3} + \ldots.$$

$$+ [2 \cdot 3 \ldots (n - 1)n + \ldots + 1 \cdot 3 \cdot 4 \ldots\ldots (n - 1)n + \ldots\ldots + 1 \cdot 2 \cdot 3 \ldots (n - 1)]\, x^1 + 1 \cdot 2 \cdot 3 \ldots n x^0$$

$\therefore \quad$ Coefficient of $x^n = 1$

Coefficient of $x^{n-1} = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

Coefficient of x^{n-2}

$$= \frac{1}{2}\left[\frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6}\right] = \frac{n(n^2-1)(3n+2)}{24}$$

Constant term $= 1 \cdot 2 \cdot 3 \cdot 4 \ldots. n = n!$

PROBLEMS

Single Option Correct Type

1. If $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots.\infty = \frac{\pi^2}{6}$ then the value of $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \ldots\infty$ equals

(a) $\frac{\pi^2}{12}$ (b) $\frac{\pi^2}{24}$ (c) $\frac{\pi^2}{48}$ (d) $\frac{\pi^2}{32}$

2. If $\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \ldots. + \frac{1}{10 \cdot 11 \cdot 12} = \frac{m}{n}$ (reduced fraction), then sum of the digits of $(m + n)$ equals

(a) 12 (b) 14 (c) 329 (d) 264

3. The natural numbers are grouped as follows {1}, {2, 3, 4}, {5, 6, 7, 8, 9}, {10, 11, 12, 13, 14, 15, 16}, ... then the mean of first term and last term of the n^{th} group is

(a) $n^2 + n + 1$ (b) $n^2 + n - 1$

(c) $n^2 - n + 1$ (d) None of these

4. The number $2^{2\sin 2x - 1}$, 3, $2^{4 - 2\sin 2x}$ form first three terms of an A.P. then its 2018th term equals

(a) 2020 (b) 2019 (c) 2018 (d) 2017

5. The coefficient of x^{16} in the product $(1 - x)(1 - 3x)(1 - 3^2x)(1 - 3^3x) \ldots\ldots(1 - 3^{16}x)$ is

(a) $3^{120}\left(\frac{3^{17}-1}{2}\right)$ (b) $3^{120}\left(\frac{3^{16}-1}{2}\right)$

(c) $3^{120}(3^{16} - 1)$ (d) $3^{120}(3^{17} - 1)$

6. Let $f_n(S) = S\,S\,S \ldots.. S$ (n digits), where $S = 1, 2, 3, \ldots, 9$ then $(f_n(6))^2 + f_n(8)$ equals to

(a) $f_{2n}(1)$ (b) $4f_{2n}(1)$

(c) $(f_{2n}(2))^2$ (d) $-4f_{2n}(1)$

7. If 4 arithmetic means, 4 geometric means and 4 harmonic means are inserted between 2 and 12 then the cubic equation whose roots are first A.M., second G.M. and third H.M. between 2 and 12 is

(a) $x^3 - (8 + 2 \cdot 6^{2/5})x^2 + 16(1 + 6^{2/5})x - 32 \cdot 6^{2/5} = 0$

(b) $x^3 + (8 + 2 \cdot 6^{2/5})x^2 + 16(1 + 6^{2/5})x + 32 \cdot 6^{2/5} = 0$

(c) $x^3 + (8 + 2 \cdot 6^{2/5})x^2 - 16(1 + 6^{2/5})x - 32 \cdot 6^{2/5} = 0$

(d) $x^3 - (8 - 2 \cdot 6^{2/5})x^2 + 16(1 - 6^{2/5})x + 32 \cdot 6^{2/5} = 0$

8. Let t_n be the n^{th} term of a G.P. of positive numbers and $\alpha = \sum_{n=1}^{2019} t_{2n}$ and $\beta = \sum_{n=1}^{2019} t_{2n-1}$ such that $\alpha \neq \beta$, then common ratio of the G.P. is

(a) $\frac{\alpha}{\beta}$ (b) $\frac{\beta}{\alpha}$ (c) $\sqrt{\frac{\beta}{\alpha}}$ (d) $\sqrt{\frac{\alpha}{\beta}}$

9. The sum of the infinite series $\frac{1 \cdot 2}{3} + \frac{2 \cdot 3}{3^2} + \frac{3 \cdot 4}{3^3} + \frac{4 \cdot 5}{3^4} + \ldots.. \infty$ is

(a) $\frac{3}{2}$ (b) $\frac{3}{4}$ (c) $\frac{9}{4}$ (d) $\frac{8}{9}$

10. The sum of infinite terms of a decreasing geometric series with common ratio 'r' is $16k$ such that $|r| < 1$, $r \neq 0$. The ratio of the 10^{th} term to the 8^{th} term is 1/25 and the ratio of the third term to the square of the second term is 1/64, then k equals to

(a) 4 (b) 5 (c) 3 (d) 2

More Than One Option(s) Correct Type

11. The number of divisors of 2014, 2015 and 2022 are in

(a) A.P. (b) G.P.

(c) A.P. and G.P. both (d) None of these

12. If $\lambda(26)^{25} = (26)^{25} + 2 \cdot 27\,(26)^{24} + 3 \cdot (27)^2(26)^{23} + \ldots. + 26(27)^{25}$ then

(a) $\sqrt{\lambda - 100} = 24$ (b) $\sqrt{\lambda - 51} = 25$

(c) $\sqrt{\lambda + 53} = 27$ (d) $\sqrt{\lambda + 165} = 29$

13. Let a, b, c, d, e be five numbers such that a, b, c are in A.P. b, c, d are in G.P. and c, d, e are in H.P. if $a = 2$, $e = 72$ then b equals

(a) 5 (b) –7 (c) 7 (d) –5

14. If three distinct numbers $a, b, c \in$ H.P. and $a^2, b^2, c^2 \in$ A.P., then the ratio $a : b : c$ is

(a) $1+\sqrt{3} : -2 : 1-\sqrt{3}$ (b) $1 : \sqrt{3} : -\sqrt{2}$

(c) $1-\sqrt{3} : -2 : 1+\sqrt{3}$ (d) $1 : -\sqrt{3} : \sqrt{2}$

15. If $a, b, c \in$ A.P. and $a^2, b^2, c^2 \in$ H.P., then

(a) a, b, c are in G.P. (b) $-\frac{a}{2}, b, c$ are in G.P.

(c) a, b, c are equal (d) $b^2 = c^2 = \frac{a^2}{2}$

16. If $S_n = 1 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots\ \forall\ n \in N$ then

(a) sum of even number of terms is negative.

(b) sum of odd number of terms is positive.

(c) $S_{100} = -5050$ (d) $S_{2n+1} < S_{2n-1}$

17. The natural numbers are divided into rows as follows.

$$\begin{array}{c} 1 \\ 2 \quad 3 \\ 4 \quad 5 \quad 6 \\ 7 \quad 8 \quad 9 \quad 10 \end{array}$$

then

(a) number of elements in the n^{th} row is n
(b) 64^{th} row ends with the number 2080
(c) most middle elements in 65^{th} row is 2113
(d) the sum of the elements of 63^{rd} row is 125055

Comprehension Type

Paragraph for Q.No. 18 to 21

Consider the sequence of integers in the form of group (1), (2, 2), (3, 3, 3), ..., $(n, n, ..., n)$ n times

18. If n occurs as 1100 term and the term is divisible by sum of the digits of n, the sum of the digits of n is

(a) 2 (b) 7 (c) 10 (d) 11

19. The sum of first 1100 terms is

(a) 31511 (b) 893 (c) 34404 (d) 35720

20. The sum of the remaining terms in the group after 1100 term in which 1100 term lies is

(a) 1316 (b) 1216 (c) 11316 (d) 11216

21. The most middle term in the group $(n, n, ..., n)$, where $n = 47$ is

(a) 1104 (b) 1105
(c) 1106 (d) none of these

Matrix Match Type

22. Match the following:

Column-I		Column-II	
P.	$\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{r}{\left(\begin{array}{l}\text{product of first }(r+1)\\ \text{positive odd integers}\end{array}\right)}$ is	1.	6
Q.	If $N=\sum\limits_{a<b}(a-b)^2$ where $a, b \in \{1, 3, 5, ..., 19\}$ then the sum of the digits of N is	2.	3
R.	The sum of infinite series $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+...$ is	3.	4
S.	The sum of infinite series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+....$ is	4.	1/2

Numerical Answer Type

23. If a function f satisfies the condition $f(x + y) = f(x)f(y)$ ∀ all natural number x and y and $f(1) = 3$. Further if k is the natural number and $\sum\limits_{k=1}^{n} f(a+k) = 243\frac{(3^n-1)}{2}$ then the value of a equals

24. If $1(2016) + 2(2015) + 3(2014) + ... + 2016(1) = 336 \times 2017 \times \alpha$, then value of α equals

25. The value of n satisfies the equation

$$2 + (6 \times 2^2 - 4 \cdot 2) + (6 \times 3^2 - 4 \cdot 3) + ... + (6 \times n^2 - 4n) = 728$$

where n is any natural number, then n equals

26. Different size of sticks are arranged in a row to form an equilateral triangle. The first row consists one stick, the second row consists two sticks and the third row consists three sticks and so on. If 771 sticks are added then all the sticks can be arranged in the shape of a square and each side of the square contains 11 sticks less than the side of the triangle. If the initial number of sticks are $39k$, then k must be

27. If $\sum\limits_{r=1}^{n} t_r = \frac{n(n+1)(n+2)}{4}$ where t_r denotes the r^{th} term of a series and $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{2019}{t_r} = 673\lambda$, then λ equals

28. The successive terms of an A.P. are $a_1, a_2, a_3, ...$. If $a_3 + a_6 + a_9 + a_{12} = 40$ and $\sum\limits_{r=1}^{14} a_r$ equals to 20λ, then λ equals

SOLUTIONS

1. **(a) :** Given, $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+....\infty=\frac{\pi^2}{6}$ ...(i)

Now, $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}-\frac{1}{6^2}+....\infty$

$$=\frac{\pi^2}{6}-2\left(\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+....\infty\right)$$

$$=\frac{\pi^2}{6}-\frac{2}{4}\left(1+\frac{1}{2^2}+\frac{1}{3^2}+....\infty\right)=\frac{\pi^2}{6}-\frac{1}{2}\times\frac{\pi^2}{6}=\frac{\pi^2}{12}$$

(Using (i))

2. **(b) :** By using V_n method, we get

$$\text{Required sum}=\frac{1}{2}\left[\frac{1}{1\cdot2}-\frac{1}{11\cdot12}\right]=\frac{1}{4}\left[1-\frac{1}{66}\right]=\frac{65}{264}=\frac{m}{n}$$

(given)

$\therefore \quad m + n = 65 + 264 = 329$

$\therefore$ Sum of digits of $m + n = 14$

3. (c) : $\because$ Last term of the 1st group is $1 = 1^2$

Last term of the 2nd group is $4 = 2^2$

Last term of the 3rd group is $9 = 3^2$

……………………………………………

……………………………………………

Last term of the $(n - 1)$th group is $(n - 1)^2$

Last term of the nth group is n^2 and the first term of this group is obtained by adding 1 in the last term of $(n - 1)$th group that is $(n - 1)^2 + 1$ *i.e.*, last group is $\{(n - 1)^2 + 1, (n - 1)^2 + 2, ..., (n - 1)^2 + 2n - 1\}$

$\therefore$ Mean value of first and last term of the nth group

$$= \frac{(n-1)^2 + 1 + n^2}{2} = n^2 - n + 1$$

4. (b) : As $2^{2\sin 2x - 1}, 3, 2^{4 - 2\sin x 2x} \in$ A.P.

$$\therefore \quad 6 = 2^{2\sin 2x - 1} + 2^{4 - 2\sin 2x}$$

$$\Rightarrow \quad 6 = \frac{2^{2\sin 2x}}{2} + \frac{16}{2^{2\sin 2x}}$$

$$\Rightarrow \quad 6 = \frac{t}{2} + \frac{16}{t} \qquad (\text{By putting } t = 2^{2\sin 2x})$$

$\Rightarrow \quad t^2 - 12t + 32 = 0 \quad \Rightarrow \quad (t - 4)(t - 8) = 0$

$\Rightarrow \quad t = 4, 8$, *i.e.*, $2^2, 2^3$

$\therefore \quad 2^{2\sin 2x} = 2^2, 2^3 \quad \Rightarrow \quad 2\sin 2x = 2, 3$

$$\therefore \quad \sin 2x = 1 \quad \left(\because \ \sin 2x = \frac{3}{2} > 1 \text{ which is not possible}\right)$$

Now, t_1 = first term $= 2^{2 - 1} = 2$; $t_2 = 3$, $t_3 = 2^2 = 4$

$\therefore$ A.P. is 2, 3, 4, ...

$\Rightarrow$ Common difference, $d = 1$

$\therefore \quad t_{2018} = a + 2017d = 2 + 2017(1) = 2019$

5. (a) : $(1 - x)(1 - 3x)(1 - 3^2x)(1 - 3^3x) \,.....\, (1 - 3^{16}x)$

$= (-1)^{17}(x - 1)(3x - 1)(3^2x - 1)(3^3x - 1) \,.....\, (3^{16}x - 1)$

$$= -3^{1+2+3+...+16} \cdot (x-1)\left(x - \frac{1}{3}\right)\left(x - \frac{1}{3^2}\right) ...... \left(x - \frac{1}{3^{16}}\right)$$

$$= -3^{\frac{16\times 17}{2}} \cdot \left\{x^{17} - \left(1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ... + \frac{1}{3^{16}}\right)x^{16} + ...\right\}$$

$\therefore$ Coefficient of x^{16}

$$= -(3^{136})\left\{(-1)\left(1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ... + \frac{1}{3^{16}}\right)\right\}$$

$$= 3^{136}\left\{\frac{1\left(1 - \left(\frac{1}{3}\right)^{17}\right)}{1 - \frac{1}{3}}\right\} = \frac{3^{137}}{2}\left(1 - \frac{1}{3^{17}}\right) = \frac{1}{2}3^{120}(3^{17} - 1)$$

6. (b) : As $f_n(S) = S\,S\,S \,.....\, S$ (n digits)

$$= S\,[111......1 \ (n \text{ digits})] = \frac{S}{9}[999\,.....\,9 \ (n \text{ digits})]$$

$$i.e., \ f_n(S) = \frac{S}{9}(10^n - 1)$$

Now, putting $S = 6$ and squaring both sides, we get

$$f_n^2(6) = \frac{36}{81}(10^n - 1)^2 = \frac{4}{9}(10^n - 1)^2 \text{ and } f_n(8) = \frac{8}{9}(10^n - 1)$$

$$\therefore \quad f_n^2(6) + f_n(8) = \frac{4}{9}(10^n - 1)^2 + \frac{8}{9}(10^n - 1)$$

$$= \frac{4}{9}(10^n - 1)[10^n - 1 + 2] = \frac{4}{9}(10^n - 1)(10^n + 1)$$

$$= \frac{4}{9}(10^{2n} - 1) = 4\left[\frac{1}{9}(10^{2n} - 1)\right] = 4 \cdot f_{2n}(1)$$

7. (a) : Let $2, a_1, a_2, a_3, a_4, 12 \in$ A.P. ....(i)

$2, g_1, g_2, g_3, g_4, 12 \in$ G.P. ....(ii)

$2, h_1, h_2, h_3, h_4, 12 \in$ H.P. ....(iii)

From (i), we have

$t_6 = 12 = 2 + 5d \ \Rightarrow \ d = 2$

$\therefore$ Terms of A.P. are 2, 4, 6, 8, 10, 12

From (ii), we have

$t_6 = 12 = 2r^5 \Rightarrow r = (6)^{1/5}$

$\therefore$ Terms G.P. of are $2, 2{\cdot}6^{1/5}, 2{\cdot}6^{2/5}, 2{\cdot}6^{3/5}, 2{\cdot}6^{4/5}, 2{\cdot}6^{5/5}$

Also, $2, h_1, h_2, h_3, h_4, 12 \in$ H.P.

$\therefore$ Terms of H.P. are

$$2, \frac{(n+1)\times a\times b}{a+nb}, \frac{(n+1)\times a\times b}{2a+(n-1)b}, \frac{(n+1)\times a\times b}{3a+(n-2)b}, \frac{(n+1)\times a\times b}{4a+(n-3)b}, 12$$

$$i.e., \ 2, \frac{5\times 2\times 12}{50}, \frac{5\times 2\times 12}{40}, \frac{5\times 2\times 12}{30}, \frac{5\times 2\times 12}{20}, 12$$

$[\because \ n = 4, a = 2, b = 12]$

$$i.e., \ 2, \frac{12}{5}, 3, 4, 6, 12$$

$\therefore$ First A.M. between 2 and 12 = 4

Second G.M. between 2 and 12 $= 2{\cdot}6^{2/5}$

Third H.M. between 2 and 12 = 4

Which are roots of the required equation.

Now, sum of roots $= 8 + 2{\cdot}6^{2/5}$

Product of roots taken two at a time

$= 16 + 16{\cdot}6^{2/5} = 16(1 + 6^{2/5})$

Product of roots taken all at a time

$= 16 \cdot 2 \cdot 6^{2/5} = 32{\cdot}6^{2/5}$

$\therefore$ Required equation is

$x^3 - (8 + 2{\cdot}6^{2/5})x^2 + 16(1 + 6^{2/5})x - 32{\cdot}6^{2/5} = 0$

8. (a) : Let a be the first term and r be the common ratio of the given G.P. then

$$\alpha = \sum_{n=1}^{2019} t_{2n} = t_2 + t_4 + t_6 + ... + t_{4038}$$

$$= ar + ar^3 + ar^5 + ... + ar^{4037}$$

$$\therefore \quad \alpha = ar(1 + r^2 + r^4 + ... + r^{4036}) \qquad ....(i)$$

$$\text{and, } \beta = \sum_{n=1}^{2019} t_{2n-1} = t_1 + t_3 + t_5 + ... + t_{4037}$$

$$= a + ar^2 + ar^4 + ... + ar^{4036}$$

$$\therefore \quad \beta = a(1 + r^2 + r^4 + ... + r^{4036}) \qquad ....(ii)$$

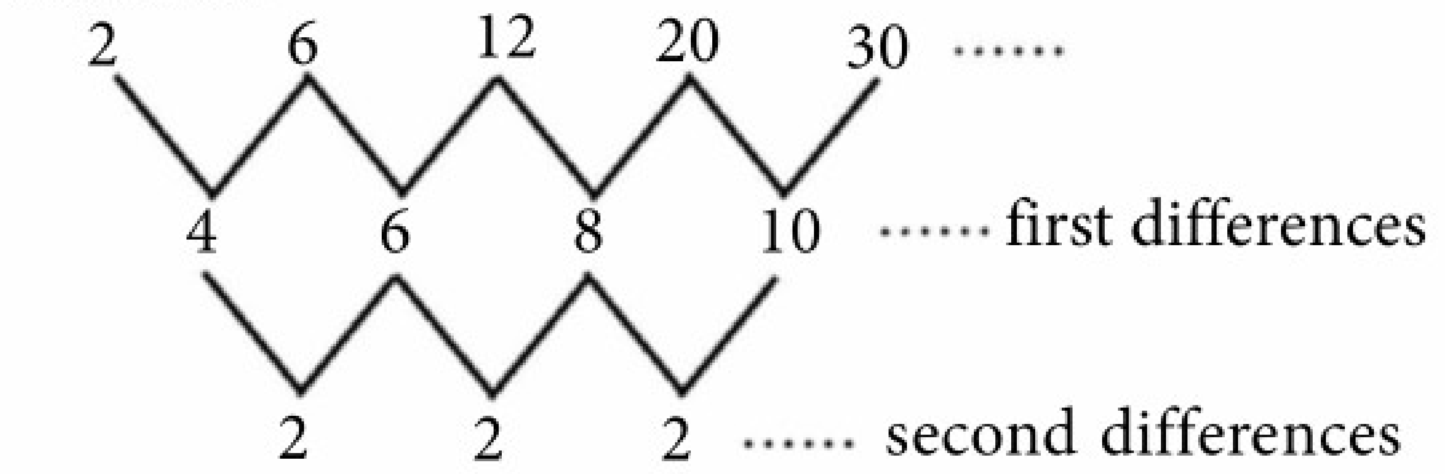
On dividing (i) and (ii), we have, $\frac{\alpha}{\beta} = r$

9. (c) : Let $S = \frac{1\cdot 2}{3} + \frac{2\cdot 3}{3^2} + \frac{3\cdot 4}{3^3} + ... \infty$

$\therefore \quad t_n = n(n+1)\cdot x^n$, where $x = \frac{1}{3}$

Now, $S = 2x + 6x^2 + 12x^3 + 20x^4 + ... \infty$

The terms in the successive order of differences of the series are


2 6 12 20 30 ......
4 6 8 10 ...... first differences
2 2 2 ...... second differences

The second order differences are equal hence the scale of relation shall be $(1 - x)^3 = 1 - 3x + 3x^2 - x^3$

$$\left.\begin{array}{rl} \text{Now, } S = & 2x + 6x^2 + 12x^3 + 20x^4 + 30x^5 + ... \\ \therefore \quad -3xS = & -6x^2 - 18x^3 - 36x^4 - 60x^5 + ... \\ 3x^2S = & 6x^3 + 18x^4 + 36x^5 + ... \\ -x^3S = & -2x^4 - 6x^5 - .... \end{array}\right] (*)$$

Adding the terms of (*) column wise, we get

$S(1 - x)^3 = 2x$ (All other terms get cancelled)

$\therefore \quad S = \frac{2x}{(1-x)^3}$; putting $x = \frac{1}{3}$, we get $S = \frac{9}{4}$

10. (b) : Let the decreasing geometric series be $a, ar, ar^2, ar^3, ...$

Now, $\frac{t_{10}}{t_8} = \frac{ar^9}{ar^7} = \frac{1}{25} \Rightarrow r^2 = \frac{1}{25} \Rightarrow r = \pm\frac{1}{5}$

$\Rightarrow \quad r = \frac{1}{5}$ (As series is decreasing geometric series and $r = -\frac{1}{5}$ is not possible)

Again, $\frac{t_3}{t_2^2} = \frac{ar^2}{(ar)^2} = \frac{1}{64} \Rightarrow \frac{1}{a} = \frac{1}{64} \Rightarrow a = 64$

$$\therefore \quad S_\infty = \frac{a}{1-r} = \frac{64}{1-\frac{1}{5}} = \frac{64\times 5}{4} = 80 = 16k \Rightarrow k = 5$$

11. (a, b, c) : We have, $2014 = 2 \times 19 \times 53$

$\therefore$ Number of divisors $= (1 + 1)(1 + 1)(1 + 1) = 8$

$2015 = 5 \times 13 \times 31$

$\therefore$ Number of divisors $= (1 + 1)(1 + 1)(1 + 1) = 8$

$2022 = 2 \times 3 \times 337$

$\therefore$ Number of divisors $= (1 + 1)(1 + 1)(1 + 1) = 8$

The number of divisors of 2014, 2015 and 2022 are 8, 8 and 8 respectively, which are in A.P. as well as in G.P.

12. (a, b, c, d) : We have $\lambda(26)^{25} = (26)^{25} + 2\cdot 27(26)^{24} + 3\cdot(27)^2(26)^{23} + ... + 26(27)^{25}$

$$\Rightarrow \quad \lambda = 1 + 2\cdot\left(\frac{27}{26}\right)^1 + 3\left(\frac{27}{26}\right)^2 + ... + 26\left(\frac{27}{26}\right)^{25}$$

$$\Rightarrow \quad \lambda = 1 + 2x + 3x^2 + ... + 26x^{25}\left(\text{where } x = \frac{27}{26}\right) \quad ...(i)$$

$$\therefore \quad \lambda x = x + 2x^2 + ... + 25x^{25} + 26x^{26} \qquad ...(ii)$$

On subtracting (ii) from (i), we get

$$\lambda(1 - x) = 1 + x + x^2 + x^3 + ... + x^{25} - 26x^{26}$$

$$\Rightarrow \quad \lambda(1 - x) = \frac{x^{26}-1}{x-1} - 26x^{26}$$

$$\Rightarrow \quad \lambda(1-x) = \frac{x^{26} - 1 - 26x^{27} + 26x^{26}}{x-1}$$

$$\Rightarrow \quad \lambda(1-x) = \frac{27x^{26} - 26x^{27} - 1}{x-1}$$

$$\Rightarrow \quad \lambda = \frac{1}{(x-1)^2} \qquad \left(\because\ 27x^{26} = 26x^{27} \text{ as } x = \frac{27}{26}\right)$$

$$\therefore \quad \lambda = (26)^2 = 676$$

$$\therefore \quad \sqrt{\lambda - 100} = \sqrt{576} = 24, \sqrt{\lambda - 51} = \sqrt{625} = 25,$$

$$\sqrt{\lambda + 53} = \sqrt{729} = 27 \text{ and } \sqrt{\lambda + 165} = \sqrt{841} = 29$$

13. (c, d) : $\because$ $a, b, c \in$ A.P. $\Rightarrow 2b = a + c$

$$\Rightarrow \quad b = \frac{2+c}{2} \qquad (\because\ a = 2)$$

$\therefore$ $b, c, d \in$ G.P. $\Rightarrow c^2 = bd$

and $c, d, e \in$ H.P.

$$\Rightarrow \quad d = \frac{2ce}{c+e} = \frac{2c\times 72}{c+72} \qquad (\because\ e = 72)$$

Now, $bd = c^2$

$$\Rightarrow \quad \frac{(2+c)}{2}\cdot\frac{(2c\times 72)}{(c+72)} = c^2 \Rightarrow 72(2+c) = c(c+72)$$

$$\Rightarrow \quad c^2 = 144 \Rightarrow c = \pm 12$$

$$\text{Now, } b = \frac{2+c}{2} \Rightarrow b = 7, -5$$

14. (a, c) : We have, $a \neq b \neq c$ and $a, b, c \in$ H.P.

$\Rightarrow \quad b = \frac{2ac}{a+c} \quad \Rightarrow \quad \frac{b}{2} = \frac{ac}{a+c} = k$ (say)

$\Rightarrow \quad b = 2k,\ ac = k(a + c)$ ....(*)

Again, $a^2, b^2, c^2 \in$ A.P.

$\Rightarrow \quad 2b^2 = a^2 + c^2 = (a + c)^2 - 2ac$

$\Rightarrow \quad 8k^2 = (a + c)^2 - 2k(a + c)$ [Using (*)]

$\Rightarrow \quad (a + c)^2 - 2(a + c)k - 8k^2 = 0$

$\Rightarrow \quad (\overline{a+c} - 4k)(\overline{a+c} + 2k) = 0 \quad \Rightarrow \quad a + c = 4k, -2k$

When $a + c = 4k$ then $ac = 4k^2$ and when $a + c = -2k$ then $ac = -2k^2$

Now, $(a - c)^2 = (a + c)^2 - 4ac = 16k^2 - 16k^2 = 0$

$\Rightarrow \quad a = c$ which contradicts the hypothesis

$\therefore \quad a + c = -2k$ and $ac = -2k^2$

Now, $(a - c)^2 = (a + c)^2 - 4ac = 4k^2 + 8k^2 = 12k^2$

$\Rightarrow \quad a - c = \pm\, 2\sqrt{3}k$

Now, $a + c = -2k$ and $a - c = \pm\, 2\sqrt{3}k$

$\therefore \quad a = (-1 \pm \sqrt{3})k$ and $c = (-1 \mp \sqrt{3})k$

$\therefore \quad a : b : c = -1 \pm \sqrt{3} : 2 : -1 \mp \sqrt{3}$

or $\quad a : b : c = 1 \mp \sqrt{3} : -2 : 1 \pm \sqrt{3}$

$\therefore \quad a : b : c = (1 - \sqrt{3} : -2 : 1 + \sqrt{3})$

or $\quad a : b : c = 1 + \sqrt{3} : -2 : 1 - \sqrt{3}$

15. (a, b, c) : $\because a, b, c \in$ A.P. $\Rightarrow \quad 2b = a + c$ ... (i)

Again, $a^2, b^2, c^2 \in$ H.P. $\Rightarrow \quad b^2 = \frac{2a^2c^2}{a^2 + c^2}$ ... (ii)

From (i) and (ii), we have, $\frac{(a+c)^2}{4} = \frac{2a^2c^2}{(c^2 + a^2)}$

$\Rightarrow \quad (a + c)^2(c^2 + a^2) - 8a^2c^2 = 0$

$\Rightarrow \quad (a^2 + c^2)^2 + 2ac(a^2 + c^2) - 8a^2c^2 = 0$

$\Rightarrow \quad (a^2 + c^2 + 4ac)(a^2 + c^2 - 2ac) = 0$

$\Rightarrow \quad a^2 + c^2 + 4ac = 0$ or $a^2 + c^2 - 2ac = 0$

$\Rightarrow \quad (a + c)^2 + 2ac = 0$ or $(a - c)^2 = 0 \Rightarrow a = c$

$\Rightarrow \quad (2b)^2 + 2ac = 0$ or $2b = 2a = 2c \quad (\because 2b = a + c)$

$\Rightarrow \quad 4b^2 = -2ac$ or $a = b = c$

$\Rightarrow \quad b^2 = \frac{-ac}{2}$ or a, b, c are both in A.P. and G.P.

$\Rightarrow \quad b^2 = \left(-\frac{a}{2}\right)(c) \quad \Rightarrow \quad -\frac{a}{2}, b, c$ are in G.P.

16. (a, b, c) : S_2 = Sum of first two terms *i.e.*, sum of even number of terms

$= 1 - 2^2 = 1 - 4 = -3 < 0 \quad \therefore$ Option (a) is correct.

S_3 = sum of first three terms *i.e.*, sum of odd number of terms

$= t_1 + t_2 + t_3 = 1 - 2^2 + 3^2 = 1 - 4 + 9 = 6 > 0$

$\therefore$ Option (b) is correct.

Also, $S_{100} = (1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + \ldots + (99^2 - 100^2)$

$= (-1)[(1 + 2) + (3 + 4) + (5 + 6) + \ldots + (99 + 100)]$

$= (-1)[1 + 2 + 3 + 4 + \ldots + 100]$

$= (-1)\frac{100 \times 101}{2} = -5050 \quad \therefore$ Option (c) is correct.

Now, S_{2n+1} and S_{2n-1} both represent the sum of odd terms and $S_1 < S_3$, $S_3 < S_5$.

Therefore, in general $S_{2n-1} < S_{2n+1}$

Thus, option (d) is not correct.

17. (a, b, c, d) : The number of elements in first, second, third row are 1, 2, 3 respectively therefore n^{th} row consists of n elements. So, option (a) is correct.

The rows are ending with 1, 3, 6, 10, ...

5th row ends with $1 + 2 + 3 + 4 + 5 = \frac{5 \times 6}{2} = 15$

$\therefore \quad n^{\text{th}}$ row ends with $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

$\therefore \quad 64^{\text{th}}$ row ends with $\frac{64 \times 65}{2} = (32 \times 65) = 2080$

Thus, option (b) is correct.

Again as 65th row having 65 elements and 33rd element is the most middle element in this row is obtained by adding 33 to the last element of 64th row, which is 2080

$\therefore \quad$ Most middle element in 65th row = 2080 + 33 = 2113 (option (c))

Now for sum of the element of 63rd row, we need either the last element of 62nd row or the first element of 63rd row. The last element of 62nd row = $\frac{62 \times 63}{2} = 1953$

$\therefore \quad$ First element of 63rd row is 1954 and last element of 63rd row = $\frac{63 \times 64}{2} = 2016$

$\therefore \quad$ Sum of the elements of 63rd row

$= \frac{n}{2}(a + l) = \frac{63}{2}(1954 + 2016) = 125055$ (Option (d))

(18-21): Deduction from the sequence of integers in the form of groups

$$(1), (2, 2), (3, 3, 3), \ldots, \underbrace{(n, n, n \ldots, n)}_{n\text{-times}}$$

- The number of digits in 1st, 2nd, 3rd, ..., n^{th} group are respectively 1, 2, 3, 4, ..., n.
- Sum of the digits of 1st, 2nd, 3rd, ..., n^{th} group are $1^2, 2^2, 3^2, \ldots, n^2$. Therefore sum of the terms upto n^{th} group is $1^2 + 2^2 + 3^2 + \ldots + n^2$.
- Sum of the different terms upto n^{th} group is $\frac{n(n+1)}{2}$, which also represent the total number of terms upto n^{th} group.

- In each group each digit is same in single digit number and in two digits it occurs in identical pair.
- The first term of $(n+1)^{\text{th}}$ group is $\left(\frac{n(n+1)}{2}+1\right)^{\text{th}}$ term.

18. (d) : Now, let 1100 term falls in n^{th} group, then

$$\frac{n(n-1)}{2}<1100\le\frac{n(n+1)}{2}$$

$\Rightarrow$ $n(n-1) < 2200$ and $n(n+1) \ge 2200$

$\Rightarrow$ $47 \times 46 = 2162 < 2200$ and $47 \times 48 = 2256 > 2200$

$\Rightarrow$ $n = 47$

$\therefore$ Sum of the digits of $n = 4 + 7 = 11$

19. (c) : Again total terms upto 46^{th} group is

$$\frac{46\times 47}{2}=1081$$

$\therefore$ Sum of first 1100 terms

$= 1^2 + 2^2 + \dots + 46^2 + (1100 - 1081) \times 47$

$= \frac{46\times 47\times 93}{6}+47\times 19 = 33511 + 893 = 34404$

20. (a) : As upto 47^{th} group the total number of terms

$= \frac{47\times 48}{2}=1128$ (each having value 47)

$\therefore$ Terms left for counting after 1100^{th} term

$= 1128 - 1100 = 28$

$\therefore$ Required sum of remaining 28 terms

$= 47 \times 28 = 1316$

21. (b) : Now, the first n occurs as 1082^{th} term and the last n occurs as 1128^{th} term in the 47^{th} group and 24^{th} forty seven is the most middle item in this group.

$\therefore$ Most middle term is $(1128 - 23)^{\text{th}}$ or $(1081 + 24)^{\text{th}}$

$= 1105^{\text{th}}$ term

22. P → 4, Q → 1, R → 1, S → 2

(P) We have,

$$t_r=\frac{r}{1\cdot 3\cdot 5\dots\cdot(2r+1)}=\frac{1}{2}\left[\frac{2r+1-1}{1\cdot 3\cdot 5\dots(2r+1)}\right]$$

$$\therefore\ t_r=\frac{1}{2}\left[\frac{1}{1\cdot 3\cdot 5\dots(2r-1)}-\frac{1}{1\cdot 3\cdot 5\dots(2r+1)}\right]$$

$$=-\frac{1}{2}[V_n-V_{n-1}] \qquad \text{(Using } V_n \text{ method)}$$

$$\therefore\ \sum_{r=1}^{n}t_r=-\frac{1}{2}[V_n-V_0]=\frac{1}{2}\left(1-\frac{1}{1\cdot 3\cdot 5\dots(2n+1)}\right)$$

$$\therefore\ \lim_{n\to\infty}S_n=\lim_{n\to\infty}\frac{1}{2}\left(1-\frac{1}{1\cdot 3\cdot 5\dots(2n+1)}\right)=\frac{1}{2}$$

(Q) $\because\ N=\sum_{a<b}(a-b)^2,\ a, b\in\{1, 3, 5, \dots, 19\},$

$$\therefore\ N=(1-3)^2+(1-5)^2+\dots+(1-19)^2$$
$$+(3-5)^2+\dots+(3-19)^2+\dots\dots+(17-19)^2$$

$$\Rightarrow\ N=9(1^2+3^2+5^2+\dots+19^2)-2\sum_{a<b}ab$$

$$=10(1^2+3^2+5^2+\dots+19^2)-(1+3+5+\dots+19)^2$$

[Using 2 (product of numbers taken two at a time) = square of the sum of numbers – sum of their squares]

$$=10\left[\frac{10\times(400-1)}{3}\right]-(10^2)^2$$

$$\left[\text{Using } 1^2+3^2+5^2+\dots+(2n-1)^2=\frac{n(4n^2-1)}{3}\right]$$

$$=\frac{100\times 399}{3}-10000 = 13300 - 10000 = 3300$$

$\therefore$ Sum of digits of $N = 3 + 3 + 0 + 0 = 6$

(R) Let $S_r=\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\dots+\frac{2r+1}{1^2+2^2+\dots+r^2}$

$$\therefore\ t_r=\frac{2r+1}{1^2+2^2+\dots+r^2}=\frac{6(2r+1)}{r(r+1)(2r+1)}$$

$$=\frac{6}{r(r+1)}=6\left[\frac{1}{r}-\frac{1}{r+1}\right]$$

$$\therefore\ \sum_{r=1}^{n}t_r=6\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)\qquad\therefore\ S_n=6\left[1-\frac{1}{n+1}\right]$$

$\therefore$ $S_\infty = 6(1 - 0) = 6$

(S) Let $S=1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\dots\infty$ ... (i)

$\therefore$ $\frac{1}{3}S=\frac{1}{3}+\frac{2}{3^2}+\frac{6}{3^3}+\frac{10}{3^4}+\dots\infty$ ... (ii)

Subtracting (ii) from (i), we get

$$\frac{2}{3}S=1+\frac{1}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\frac{4}{3^4}+\dots\infty$$

$$=\frac{4}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\dots\infty$$

$$=\frac{4}{3}\left(1+\frac{1}{3}+\frac{1}{3^2}+\dots\infty\right)=\frac{4}{3}\times\frac{1}{\left(1-\frac{1}{3}\right)}=\frac{4}{3}\times\frac{3}{2}=2$$

$$\therefore\ S=\frac{2\times 3}{2}=3$$

23. (4) : We have, $f(1) = 3$ and $f(x + y) = f(x) \cdot f(y)$.

Putting $x = 1$, $y = 1$, we get

$f(1 + 1) = f(1)\, f(1) = [f(1)]^2 \Rightarrow f(2) = 3^2$

Also, $f(1+2) = f(1)\,f(2) \Rightarrow f(3) = 3^3$
And $f(1+3) = f(1)\,f(3) = 3\cdot 3^3 = 3^4 \Rightarrow f(4) = 3^4$
....
....
Similarly, $f(k) = 3^k$ and $f(a) = 3^a$ .... (*)

Now, $\sum_{k=1}^{n} f(a+k) = \sum_{k=1}^{n} f(a)\cdot f(k)$

$$= f(a)\sum_{k=1}^{n} f(k) = f(a)\sum_{k=1}^{n} 3^k$$

$$= f(a)[3^1 + 3^2 + 3^3 + \ldots + 3^n]$$

$$\therefore \sum_{k=1}^{n} f(a+k) = 3f(a)\frac{(3^n-1)}{2}$$

$$\Rightarrow 243\frac{(3^n-1)}{2} = 3f(a)\left(\frac{3^n-1}{2}\right)$$

$$\Rightarrow 81 = f(a) = 3^a \Rightarrow 3^4 = 3^a \Rightarrow a = 4$$

24. (2018) : ∵ $1(2016) + 2(2015) + 3(2014) + \ldots + 2016(1)$

$$= \sum_{r=1}^{2016} r(2016-(r-1)) = \sum_{r=1}^{2016} r(2017-r)$$

$$= \sum_{r=1}^{2016} 2017r - \sum_{r=1}^{2016} r^2$$

$$= \frac{2017\times 2016\times 2017}{2} - \frac{2016\times 2017\times 4033}{6}$$

$$= \frac{2017\times 2016}{6}(6051 - 4033) = 2017\times 336\times 2018$$

$$= 2017\times 336\times \alpha \Rightarrow \alpha = 2018$$

25. (7) : Let $S = 2 + (6\times 2^2 - 4\cdot 2) + (6\times 3^2 - 4\cdot 3) + \ldots + (6\times n^2 - 4n)$

$\therefore\ S = 2 + 6(2^2 + 3^2 + \ldots + n^2) - 4(2 + 3 + 4 + \ldots + n)$
$= 2 + 6(1^2 + 2^2 + \ldots + n^2) - 4(1 + 2 + 3 + \ldots + n) - 6 + 4$
$= 6(1^2 + 2^2 + \ldots + n^2) - 4(1 + 2 + 3 + \ldots + n)$

$$= 6\cdot\frac{n(n+1)(2n+1)}{6} - 4\frac{n(n+1)}{2}$$

$= n(n+1)[2n+1-2] = n(n+1)(2n-1)$
$\therefore\ n(n+1)(2n-1) = 728 = 7\times 8\times 13$
$= 7\times(7+1)(2\times 7-1) \Rightarrow n = 7$

26. (55) : As the first row contains 1 stick, 2nd row contains 2 sticks and third row contains 3 sticks, and so on.

$\therefore$ Initially the number of sticks $= 1 + 2 + 3 + \ldots + n$

$$= \frac{n(n+1)}{2} \quad \ldots \text{(i)}$$

According to problem, we have

$$\frac{n(n+1)}{2} + 771 = (n-11)^2$$

$$\Rightarrow \frac{n^2+n+1542}{2} = n^2 - 22n + 121$$

$\Rightarrow\ n^2 + n + 1542 = 2n^2 - 44n + 242$
$\Rightarrow\ n^2 - 45n - 1300 = 0$
$\Rightarrow\ (n-65)(n+20) = 0 \Rightarrow n = 65,\ [\because n \neq -20]$
$\therefore$ Initially the number of sticks

$$= \frac{n(n+1)}{2} = \frac{65\times 66}{2} = 2145$$

$\because\ 39k = 2145 = 39\times 55 \Rightarrow k = 55$

27. (4) : We have, $\sum_{r=1}^{n} t_r = \frac{n(n+1)(n+2)}{4} = S_n(\text{say})$

$$\therefore \sum_{r=1}^{n-1} t_r = \frac{(n-1)(n)(n+1)}{4} = S_{n-1}$$

$$\therefore t_n = \frac{n(n+1)(n+2)}{4} - \frac{(n-1)(n)(n+1)}{4}$$

$$= \frac{n(n+1)}{4}[n+2-n+1] = \frac{3n(n+1)}{4}$$

$$\therefore \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{t_r} = \frac{4}{3}\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{r(r+1)}$$

$$= \frac{4}{3}\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right) \quad \text{(Using partial fraction)}$$

$$= \frac{4}{3}\lim_{n\to\infty}\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\ldots+\left(\frac{1}{n-1}-\frac{1}{n}\right)+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right]$$

$$= \frac{4}{3}\lim_{n\to\infty}\left(1-\frac{1}{n+1}\right) = \frac{4}{3}(1-0) = \frac{4}{3}$$

$$\therefore \lim_{n\to\infty}\sum_{r=1}^{n}\frac{2019}{t_r} = \frac{4}{3}\times 2019 = 673\lambda \Rightarrow \lambda = 4$$

28. (7) : Let the common difference of A.P. is d and $a_1 = a$ be the first term, then series becomes a, $a+d$, $a+2d$, ...

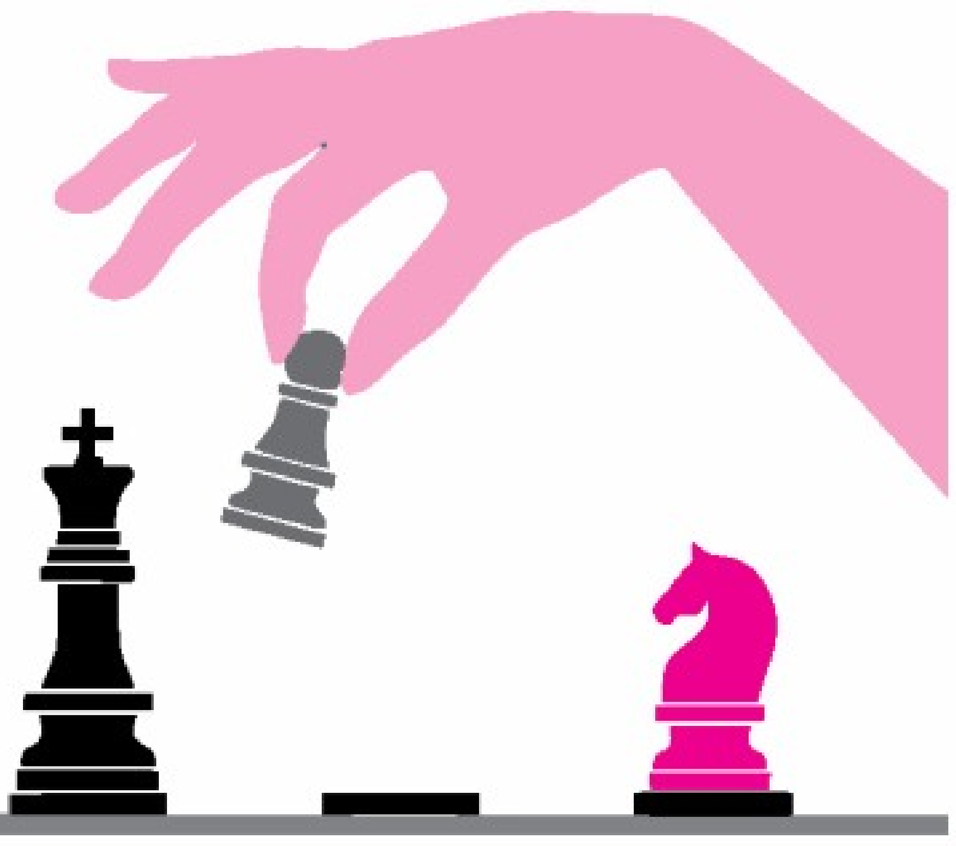
Now, $a_3 + a_6 + a_9 + a_{12} = 40$
$\Rightarrow\ (a+2d) + (a+5d) + (a+8d) + (a+11d) = 40$
$\Rightarrow\ 4a + 26d = 40 \Rightarrow 2a + 13d = 20$

$$\therefore \sum_{r=1}^{14} a_r = S_{14} = \frac{14}{2}[2a+13d] = 7\times 20 = 140$$

$$\because \sum_{r=1}^{14} a_r = 20\lambda \Rightarrow 140 = 20\lambda \Rightarrow \lambda = 7$$

◆◆

Challenging PROBLEMS ON Integral Calculus



Passage (1-3) :

If $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}} = A\sin^{-1}\{\sqrt{f(x)}\}+c$, then

1. The value of A is

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) $-\frac{1}{\sqrt{2}}$ (d) None of these

2. The $f(x)$ is

(a) $\frac{1+x^2}{1-x^2}$ (b) $\frac{1-x^2}{1+x^2}$

(c) $\sqrt{\frac{1-x^2}{1+x^2}}$ (d) $\sqrt{1-x^2}$

3. $\int f(x)dx$ is

(a) $2\tan^{-1}x - x + c$ (b) $2\tan^{-1}x + x + c$

(c) $\tan^{-1}x + 2x + c$ (d) none of these

Passage (4-6): Let $I_n = \int \tan^n x\,dx,\ I_n' = \int \cot^n x\,dx, n \in N$

$f(x,n) = I_n + I_{n-2}(n \neq 1), g(x,n) = I_n' + I_{n-2}'(n \neq 1),$

$f(0,n) = g\left(\frac{\pi}{2}, n\right) = 0$

4. $\lim\limits_{x\to 0} \frac{f(x,2n+1)}{x^{2n}g(x,2n+1)} =$

(a) 0 (b) 1 (c) -1 (d) 2

5. If $I_0 + I_1 + 2(I_2 + I_3 + \dots\dots + I_{2006} + I_{2009}) + 3I_{2007} + I_{2008}$

$= \sum_{r=1}^{2008} a_r \tan^r x$, then $a_{2008} =$

(a) 1/2007 (b) 2/2007

(c) 1/1004 (d) 1/2008

6. If $I_0' + I_1' + 2(I_2' + I_3' + \dots\dots + I_{18}') + I_{19}' + I_{20}'$

$= \sum_{r=1}^{19} a_r \cot^r x$, then $a_{10} =$

(a) 1/10 (b) $-1/10$ (c) 1/9 (d) $-2/9$

7. Area enclosed by the curve $x = a\cos^3 t$, $y = a\sin^3 t$, $0 \le t \le 2\pi$ is

(a) $\frac{3}{2}a^2\pi$ (b) $\frac{3}{4}a^2\pi$ (c) $\frac{3}{8}a^2\pi$ (d) $\frac{3}{16}a^2\pi$

Passage (8-9) : Let m, n be two positive real numbers and define

$$f(n) = \int_0^\infty x^{n-1}e^{-x}dx \text{ and } g(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$$

It is known that $f(n)$, for $n > 0$ is finite and $g(m, n) = g(n, m)$ for $m, n > 0$.

8. $\int_0^1 x^m \left(\log_e \frac{1}{x}\right)^n dx =$

(a) $\frac{f(n+1)}{(m+1)^n}$ (b) $\frac{f(n)}{(m+1)^{n+1}}$

(c) $\frac{f(n+1)}{(m+1)^{n+1}}$ (d) $g(m + 1, n + 1)$

9. $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx =$

(a) $g(n, m)$ (b) $g(m - 1, n + 1)$

(c) $g(m - 1, n - 1)$ (d) $g(m + 1, n - 1)$

10. The value of $\int_0^{\pi/2} \log(\sin^2\theta + k^2\cos^2\theta)d\theta$, where $k \ge 0$, is

(a) $\pi\log(1 + k) + \pi\log 2$ (b) $\pi\log(1 + k)$

(c) $\pi\log(1 + k) - \pi\log 2$ (d) $\pi\log(1 + k) - \log 2$

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11. If $\int_0^{\pi} \frac{dx}{(a-\cos x)} = \frac{\pi}{\sqrt{a^2-1}}$, then the value of $\int_0^{\pi} \frac{dx}{(\sqrt{10}-\cos x)^3}$ is

(a) $\frac{\pi}{81}$ (b) $\frac{7\pi}{81}$

(c) $\frac{7\pi}{162}$ (d) None of these

12. If $I = \int_0^{\pi} x(\sin^2(\sin x)+\cos^2(\cos x))dx$, then $[I]$ = ______, where $[\cdot]$ denotes the greatest integer function.

(a) 4 (b) 3 (c) 2 (d) 1

13. Let $f(x) = x^3 + 3x + 2$ and $g(x)$ is the inverse of it. The area bounded by $g(x)$, the x-axis and the ordinates at $x = -2$ and $x = 6$ is $\frac{m}{n}$ where $m, n \in N$ and G.C.D. of $(m, n) = 1$ then $m - n =$

(a) 6 (b) 7 (c) 8 (d) 9

14. Given that $\int_1^y \sec^{-1} x\, dx = \lambda$ and

$$\int_{-y}^{-1}\left[\sec^{-1} x - \tan^{-1}\left(\sqrt{x^2-1}\right)\right]dx +$$

$$\int_1^y\left[\sec^{-1} x - \tan^{-1}\left(\sqrt{x^2-1}\right)\right]dx, (|y| \geq 1)$$

equals to $\pi(y - a) - b\lambda$, then $a + b =$

(a) 1 (b) 2 (c) 3 (d) 4

15. Find the value of $\dfrac{29\int_0^1 (1-x^4)^7\,dx}{4\int_0^1 (1-x^4)^6\,dx}$.

(a) 3 (b) 3/7 (c) 7/3 (d) 7

16. If $\int \frac{ax^2+2bx+c}{(Ax^2+2Bx+C)^2}dx\,(B^2 \neq AC)$ is a rational function of x then which is necessarily true ?

(a) $2Bb = Ac + aC$ (b) $2Bb = Aa + cC$

(c) $2Cc = Ab + Ba$ (d) $2aA = bB + cC$

17. Let $S_n = \sum_{k=1}^{n}\left(\frac{(-1)^{k-1}}{k}\,{}^nC_k\right)$, if $[\cdot]$ denotes G.I.F. then $\left[\frac{1}{2(S_{10}-S_9)}\right] =$

(a) 5 (b) 6 (c) 7 (d) 9

18. Value of 'k' so that $f(k) = \int_0^4 |4x - x^2 - k|\,dx$ is minimum is

(a) 1 (b) 2 (c) 3 (d) 4

SOLUTIONS

(1-3) : Let $I = \int \frac{dx}{(1+x^2)(\sqrt{1-x^2})} = \int \frac{dx}{x^3\left(1+\frac{1}{x^2}\right)\sqrt{\frac{1}{x^2}-1}}$

Put $\frac{1}{x^2} - 1 = t^2 \Rightarrow -\frac{2}{x^3}dx = 2t\,dt$

$\therefore\ I = -\int \frac{tdt}{(t^2+2)t} = -\frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}} + c$

$= -\frac{1}{\sqrt{2}}\tan^{-1}\frac{\sqrt{1-x^2}}{\sqrt{2}x} + c = -\frac{1}{\sqrt{2}}\sin^{-1}\sqrt{\frac{1-x^2}{1+x^2}} + c$

$\therefore\ A = -\frac{1}{\sqrt{2}}, f(x) = \frac{1-x^2}{1+x^2}$

Now, $\int f(x)\,dx = \int \frac{1-x^2}{1+x^2}dx = \int\left(\frac{2}{1+x^2} - 1\right)dx$

$= 2\tan^{-1}x - x + c$

1. (c) **2. (b)** **3. (a)**

4. (a) : $f(x, 2n+1) = \frac{\tan^{2n} x}{2n}, g(x, 2n+1) = -\frac{\cot^{2n} x}{2n}$

5. (c) : $I_n + I_{n-2} = \frac{\tan^{n-1} x}{n-1}$

6. (b) : $I_n' + I_{n-2}' = -\frac{\cot^{n-1} x}{n-1}$

7. (c) : $x = a\cos^3 t,\ y = a\sin^3 t,\ 0 \leq t \leq 2\pi$

Let $f(t) = a\cos^3 t$ and $g(t) = a\sin^3 t$

$\therefore\ (xg'(t) - yf'(t)) = a^2(\cos^3 t \cdot 3\sin^2 t \cdot \cos t + \sin^3 t \cdot 3\cos^2 t \cdot \sin t) = 3a^2\cos^2 t \cdot \sin^2 t$

$\Rightarrow$ Area of the curve $= \frac{1}{2}\int_0^{2\pi}(xg'(t) - yf'(t))dt$

$= \frac{1}{2}\int_0^{2\pi} 3a^2\cos^2 t\sin^2 t\,dt = \frac{3}{8}a^2\pi$

8. (c) : Let $I = \int_0^1 x^m\left(\log_e\frac{1}{x}\right)^n dx$

Put $\log_e\frac{1}{x} = t \Rightarrow x = e^{-t} \Rightarrow dx = -e^{-t}dt$

$\therefore\ I = \int_{\infty}^{0} e^{-mt}t^n(-e^{-t})dt$

$= \int_0^{\infty} t^n e^{-(m+1)t}dt = \frac{1}{(m+1)^{n+1}}\int_0^{\infty} t^n e^{-t}dt = \frac{f(n+1)}{(m+1)^{n+1}}$

9. (a) : Let

$$I=\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx=\int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}}dx+\int_0^1 \frac{x^{n-1}}{(1+x)^{m+n}}dx$$

$= I_1 + I_2$

In I_2, put $x=\frac{1}{t}$ then $I_2=\int_\infty^1 \frac{\frac{1}{t^{n-1}}}{\left(1+\frac{1}{t}\right)^{m+n}}\left(\frac{-dt}{t^2}\right)$

$$=\int_1^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx$$

$$\therefore \quad I=\int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}}dx+\int_1^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx$$

$$=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx=g(m,n)$$

10. (c) : Let $f(k)=\int_0^{\pi/2} \log(\sin^2\theta+k^2\cos^2\theta)d\theta$

$\therefore \quad f'(k)=\frac{\pi}{1+k}$ and $f(1)=0$

$\Rightarrow \quad f(k) = \pi\log(1 + k) - \pi\log 2$

11. (c) : We have, $\int_0^\pi \frac{dx}{(a-\cos x)}=\frac{\pi}{\sqrt{a^2-1}}$

Diff. w.r.t. a on both sides treating x as constant, we get

$$-\int_0^\pi \frac{dx}{(a-\cos x)^2}=\frac{-\pi a}{(a^2-1)^{3/2}}$$

Again diff. w.r.t. a treating x as constant, we get

$$2\int_0^\pi \frac{dx}{(a-\cos x)^3}=\frac{\pi(1+2a^2)}{(a^2-1)^{5/2}}$$

12. (a) : $I=\int_0^\pi (\pi-x)((\sin^2(\sin x))+\cos^2(\cos x))dx$

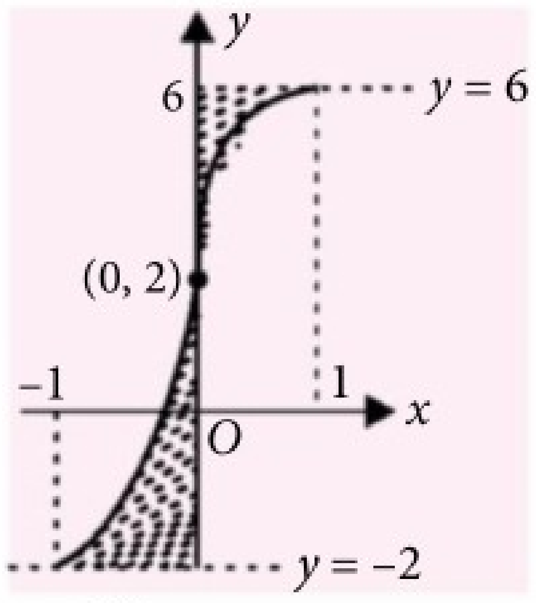
$$\Rightarrow \quad 2I=\pi\int_0^\pi (\sin^2(\sin x)+\cos^2(\cos x))dx$$

$$\Rightarrow \quad I=\pi\int_0^{\pi/2} (\sin^2(\sin x)+\cos^2(\cos x))dx$$

$$=\pi\int_0^{\pi/2} (\sin^2(\cos x)+\cos^2(\sin x))dx$$

$$\Rightarrow \quad 2I=\pi\int_0^{\pi/2} 2dx \;\Rightarrow\; I=\frac{\pi^2}{2}$$

13. (b) : The required area will be equal to area enclosed by $y = f(x)$, the y-axis and the abscissa at $y = -2$ and $y = 6$.



Required area

$$=\int_0^1 [6-f(x)]\,dx$$

$$+\int_{-1}^0 [f(x)-(-2)]dx$$

$$=\frac{9}{2}=\frac{m}{n} \Rightarrow m-n=7$$

14. (c) : $\int_{-y}^{y}\left[\sec^{-1}x-\tan^{-1}\left(\sqrt{x^2-1}\right)\right]dx$

$$=\int_{-y}^{-1}[\sec^{-1}x-(\pi-\sec^{-1}x)]dx+\int_1^y[\sec^{-1}x-\sec^{-1}x]dx$$

$$=2\int_{-y}^{-1}\sec^{-1}x\,dx-\pi\int_{-y}^{-1}dx$$

$$=2\int_1^y(\pi-\sec^{-1}x)dx-\pi(y-1)=\pi(y-1)-2\lambda$$

$\therefore \quad a + b = 3$

15. (d) : Let $I=\int_0^1 (1-x^4)^7(1)dx$

$$\Rightarrow \quad I=\left[x(1-x^4)^7\right]_0^1+7\times 4\int_0^1 x(1-x^4)^6x^3dx$$

$$\Rightarrow \quad I=28\int_0^1 (1-x^4)^6dx-28\int_0^1 (1-x^4)^7dx$$

$$=28\int_0^1 (1-x^4)^6dx-28I \Rightarrow \quad 29I=28\int_0^1 (1-x^4)^6dx$$

$$\therefore \quad \frac{29\int_0^1 (1-x^4)^7dx}{4\int_0^1 (1-x^4)^6dx}=7$$

16. (a) : Write given integral as $\frac{p(x)}{q(x)}$ and then differentiate on both sides and compare.

17. (a) : $\frac{1}{k}=\int_0^1 x^{k-1}dx$

$$\Rightarrow \quad S_{10}=\sum_{k=1}^{10}(-1)^{k-1}\left[\int_0^1 x^{k-1}dx\right]{}^{10}C_k$$

$$=\int_0^1 \frac{(1-x)^{10}-1}{(1-x)-1}dx=\int_0^1 \frac{x^{10}-1}{x-1}dx$$

$$S_9=\int_0^1 \frac{x^9-1}{x-1}dx \;\Rightarrow\; S_{10}-S_9=\frac{1}{10}$$

18. (c) : We have to minimize the area enclosed between $y = 4x - x^2$, $y = k$, $x = 0$, $x = 4$

Since, $y = 4x - x^2$ is symmetric about $x = 2$.

So, minimum value occurs at 1 in [0, 2].

Hence, $k = 3$

❖❖

MATH archives

10 BEST PROBLEMS
$(x+y)^2 = x^2$

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE (Main & Advanced) Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE (Main & Advanced). In every issue of MT, challenging problems are offered with detailed solution. The reader's comments and suggestions regarding the problems and solutions offered are always welcome.

1. For a binomial distribution, mean is 6 and S.D. is $\sqrt{2}$. The distribution is

(a) $\left(\frac{2}{3}+\frac{1}{3}\right)^9$ (b) $\left(\frac{1}{3}+\frac{2}{3}\right)^9$

(c) $\left(\frac{1}{5}+\frac{4}{5}\right)^9$ (d) $\left(\frac{4}{5}+\frac{1}{5}\right)^9$

2. A student is given 6 questions in a true or false examination. If he gets 4 or more correct answers he passes the examination. The probability that he passes the examination is

(a) $\frac{5}{32}$ (b) $\frac{7}{32}$ (c) $\frac{11}{32}$ (d) $\frac{3}{32}$

3. 5 cards are drawn one after another successively with replacement from a well shuffled pack of 52 cards. The probability that all the 5 cards are spades is

(a) $\left(\frac{3}{4}\right)^5$ (b) $1-\left(\frac{3}{4}\right)^5$

(c) $\left(\frac{1}{4}\right)^5$ (d) $1-\left(\frac{1}{4}\right)^5$

4. Let H be the set of all houses in a city where each house is faced in one of the directions East, West, North, South.

Let $R = \{(x, y) : (x, y) \in H \times H$ and x, y are faced in same direction$\}$. Then the relation R is

(a) Not reflexive, symmetric and transitive

(b) Reflexive, symmetric, not transitive

(c) Symmetric, not reflexive, not transitive

(d) An equivalence relation

5. If $10^n + 3\cdot 4^{n+2} + k$ is divisible by 9, for all $n \in N$, then the least positive integral value of k is

(a) 5 (b) 3 (c) 7 (d) 1

6. For all $n \in N$, $\{3(5^{2n+1}) + 2^{3n+1}\}$ is divisible by k, where k is prime. Then the least prime greater than k, is

(a) 13 (b) 17 (c) 19 (d) 23

7. The top of a hill observed from the top and bottom of a building of height 'h' is at angles of elevation 'p' and 'q' respectively. The height of the hill is

(a) $\frac{h\cot q}{\cot q-\cot p}$ (b) $\frac{h\cot p}{\cot p-\cot q}$

(c) $\frac{h\tan p}{\tan p-\tan q}$ (d) $\frac{h\tan p}{\cot q+\cot p}$

8. An aeroplane flying at a height of 3000 m above the ground passes vertically above another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. The height of the lower plane from the ground is

(a) $1000\sqrt{3}$ m (b) $(1000/\sqrt{3})$ m

(c) 500 m (d) $1500(\sqrt{3}+1)$ m

9. A man on the top of a cliff 100 m high, observes the angles of depression of two points on the opposite sides of the cliff as 30° and 60° respectively. Then the distance between the two points is equal to

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(a) $400\sqrt{3}$ m (b) $\frac{400}{\sqrt{3}}$ m

(c) $\frac{100}{\sqrt{3}}$ m (d) $\frac{200}{\sqrt{3}}$ m

10. AB is a vertical pole. The end A is on the level ground. C is the middle point of AB. P is a point on the level ground. The portion BC subtends an angle β at P. If $AP = 5AB$, then $\tan\beta =$

(a) $\frac{5}{51}$ (b) $\frac{5}{24}$ (c) $\frac{5}{26}$ (d) $\frac{5}{28}$

SOLUTIONS

1. (b) : We have, $np = 6$, S.D. $= \sqrt{2} \Rightarrow npq = 2$

$\therefore \frac{npq}{np} = \frac{2}{6} \Rightarrow q = \frac{1}{3}$

Now, $p + q = 1 \Rightarrow p = 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore np = 6 \Rightarrow n = 6 \times \frac{3}{2} = 9$

Thus, distribution is $\left(\frac{1}{3} + \frac{2}{3}\right)^9$.

2. (c) : The student passess the examination if gets 4 or more correct answers.

Now, $P(\text{correct answer}) = \frac{1}{2}$

Here, $n = 6$, $p = \frac{1}{2}, q = \frac{1}{2}$

$\therefore$ Required probability

$$= {}^6C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2 + {}^6C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right) + {}^6C_6\left(\frac{1}{2}\right)^6$$

$$= 15\left(\frac{1}{2}\right)^6 + 6\left(\frac{1}{2}\right)^6 + 1\left(\frac{1}{2}\right)^6 = \frac{22}{2^6} = \frac{11}{2^5} = \frac{11}{32}$$

3. (c) : We have, $P(\text{drawn card is of spade}) = \frac{13}{52} = \frac{1}{4}$

Here, $n = 5$, $p = \frac{1}{4}, q = \frac{3}{4}$

$\therefore$ Required probability $= {}^5C_5\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0 = \left(\frac{1}{4}\right)^5$

4. (d) : We have, $R = \{(x, y) : (x, y) \in H \times H$ and x, y are faced in same direction$\}$

Reflexive : $(x, x) \in H \times H \ \forall \ x \in H$.

$\Rightarrow (x, x) \in R \Rightarrow R$ is reflexive.

Symmetric : Let $(x, y) \in R$

$\Rightarrow$ x, y are faced in same direction

$\Rightarrow$ y, x are faced in same direction

$\Rightarrow (y, x) \in R$.

So, $(x, y) \in R \Rightarrow (y, x) \in R \Rightarrow R$ is symmetric.

Transitive : Let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow$ x, y are faced in same direction and y, z are faced in same direction.

$\Rightarrow$ x, z are faced in same direction.

$\Rightarrow (x, z) \in R$

So, $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$

$\Rightarrow$ R is transitive

Thus, R is an equivalence relation.

5. (a) : Let $P(n) = 10^n + 3.4^{n+2} + k$, which is divisible by 9.

$\therefore$ $P(1) = 10 + 3.4^3 + k$, which is divisible by 9.

$= 10 + 192 + k$, which is divisible by 9

$= 202 + k$, which is divisible by 9

$\therefore$ $P(1)$ will be divisible by 9 for least positive integral value of $k = 5$.

6. (c) : Let $P(n) = 3(5^{2n+1}) + 2^{3n+1}$, which is divisible by k.

$\therefore$ $P(1) = 3(5^3) + 2^4 = 375 + 16 = 391$

$= 17 \times 23$, which is divisible by both 17 and 23

$\Rightarrow k = 17$ or 23

Thus, the least prime greater than k is 19.

SAMURAI SUDOKU

ANSWER - SEPTEMBER 2019

4	7	6	2	1	8	3	9	5				7	5	2	1	4	9	8	6	3
5	8	9	3	7	6	4	1	2				3	9	4	6	8	5	7	1	2
1	3	2	4	9	5	7	8	6				1	6	8	3	7	2	4	5	9
2	9	3	8	6	7	1	5	4				2	3	1	7	6	4	5	9	8
8	6	1	5	2	4	9	7	3				4	7	5	9	3	8	6	2	1
7	4	5	1	3	9	6	2	8				9	8	6	5	2	1	3	7	4
3	1	4	7	5	2	8	6	9	2	4	3	5	1	7	8	9	3	2	4	6
6	2	7	9	8	3	5	4	1	7	8	9	6	2	3	4	1	7	9	8	5
9	5	8	6	4	1	2	3	7	6	5	1	8	4	9	2	5	6	1	3	7
						1	8	6	5	3	7	2	9	4						
						4	2	3	1	9	8	7	5	6						
						7	9	5	4	6	2	1	3	8						
5	9	6	4	2	7	3	1	8	9	2	6	4	7	5	6	8	9	2	1	3
3	2	8	6	5	1	9	7	4	8	1	5	3	6	2	1	4	5	7	8	9
7	1	4	8	3	9	6	5	2	3	7	4	9	8	1	3	2	7	6	5	4
4	5	1	9	6	8	7	2	3				6	5	3	2	7	4	8	9	1
9	7	3	1	4	2	5	8	6				2	9	4	8	6	1	3	7	5
6	8	2	3	7	5	1	4	9				7	1	8	9	5	3	4	6	2
1	6	5	2	8	3	4	9	7				1	3	7	4	9	6	5	2	8
2	4	9	7	1	6	8	3	5				5	2	9	7	3	8	1	4	6
8	3	7	5	9	4	2	6	1				8	4	6	5	1	2	9	3	7

7. (b) : Let AD be the building of height 'h' and BP be the hill.

In ΔPAB,

$$\tan q = \frac{h+x}{y} \qquad \text{...(i)}$$

And in ΔPDC, $\tan p = \frac{x}{y}$

$\Rightarrow \quad y = x \cot p \qquad$ ...(ii)

From (i) and (ii), we get

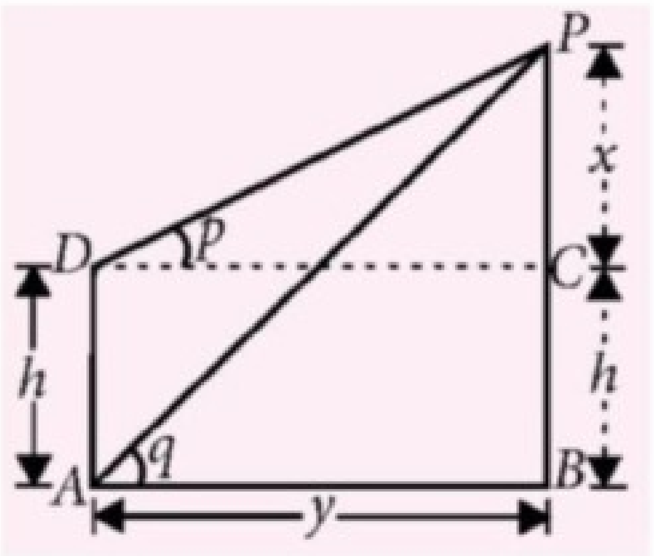
$$\tan q = \frac{h+x}{x \cot p}$$

$\Rightarrow \quad x \cot p = (h + x)\cot q$

$$\Rightarrow \quad x = \frac{h \cot q}{\cot p - \cot q}$$

$$\Rightarrow \quad h + x = \frac{h \cot q}{\cot p - \cot q} + h$$

$$\therefore \quad \text{Height of the hill} = \frac{h \cot p}{\cot p - \cot q}$$



8. (a) : Let P and Q be the position of the two planes and A be the point on the ground such that $\angle QAO = 60°$, $\angle PAO = 45°$

Let OP, the height of lower plane from the ground be h.

Also, $OQ = 3000$ m

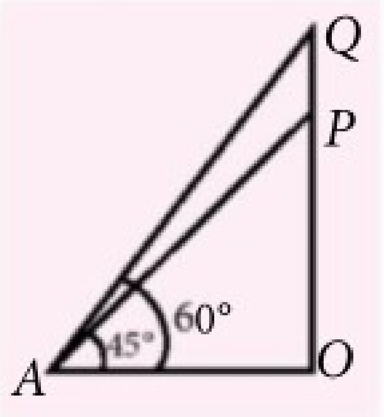
Then, in ΔPAO, $\tan 45° = \frac{OP}{OA} \Rightarrow OA = OP = h$

And, in ΔAOQ, $\tan 60° = \frac{OQ}{OA}$

$$\Rightarrow \quad \tan 60° = \frac{3000}{h}$$

$\Rightarrow \quad h \tan 60° = 3000$

$$\Rightarrow \quad h = 3000 \cot 60° = \frac{3000}{\sqrt{3}} = 1000\sqrt{3}\text{ m}$$



9. (b) : Let PQ be the cliff and A and B be the points under observation.

Now, $PQ = 100$ m

In ΔAPQ, $\tan 30° = \frac{PQ}{AP}$

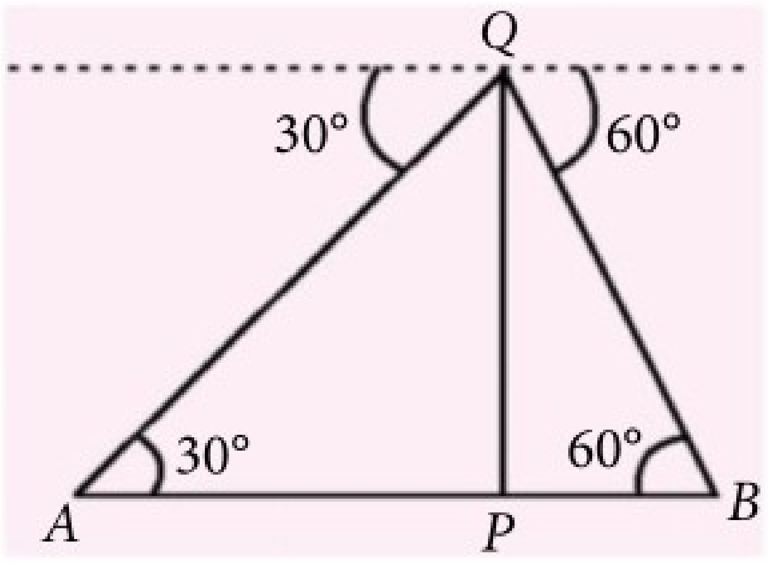
$\Rightarrow AP = 100 \cot 30°$

$\quad = 100\sqrt{3}$ m

In ΔBPQ, $\tan 60° = \frac{PQ}{BP}$

$$\Rightarrow \quad BP = 100 \cot 60° = \frac{100}{\sqrt{3}}\text{ m}$$

$$\therefore \quad AB = AP + PB = 100\sqrt{3} + \frac{100}{\sqrt{3}} = \frac{300+100}{\sqrt{3}} = \frac{400}{\sqrt{3}}\text{ m}$$



10. (a) : We have, $\angle BPC = \beta$

Let $\angle CPA = \alpha$

In ΔCPA, $\tan\alpha = \frac{AC}{AP}$

$\Rightarrow \quad AC = AP \tan\alpha$

$$\Rightarrow \quad \frac{1}{2}AB = 5AB \tan\alpha$$

[$\because$ $AC = BC$ and $AP = 5AB$ (Given)]

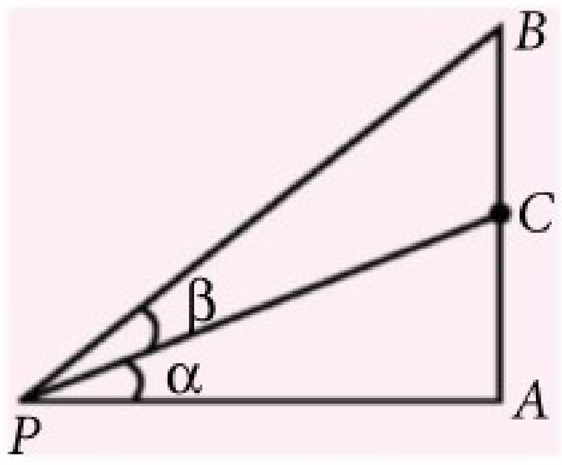
$$\Rightarrow \quad \tan\alpha = \frac{1}{10}$$

In ΔBPA, $\tan(\alpha+\beta) = \frac{AB}{AP} = \frac{1}{5}$

$$\Rightarrow \quad \frac{\frac{1}{10} + \tan\beta}{1 - \frac{1}{10}\tan\beta} = \frac{1}{5} \quad \Rightarrow \quad \frac{5}{10} + 5\tan\beta = 1 - \frac{1}{10}\tan\beta$$

$$\Rightarrow \quad \left(5 + \frac{1}{10}\right)\tan\beta = 1 - \frac{1}{2} \Rightarrow \frac{51}{10}\tan\beta = \frac{1}{2}$$

$$\Rightarrow \quad \tan\beta = \frac{5}{51}$$



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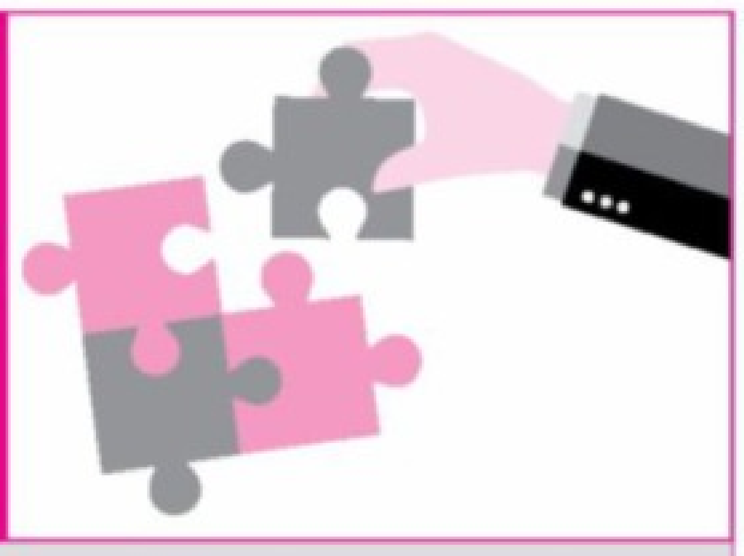
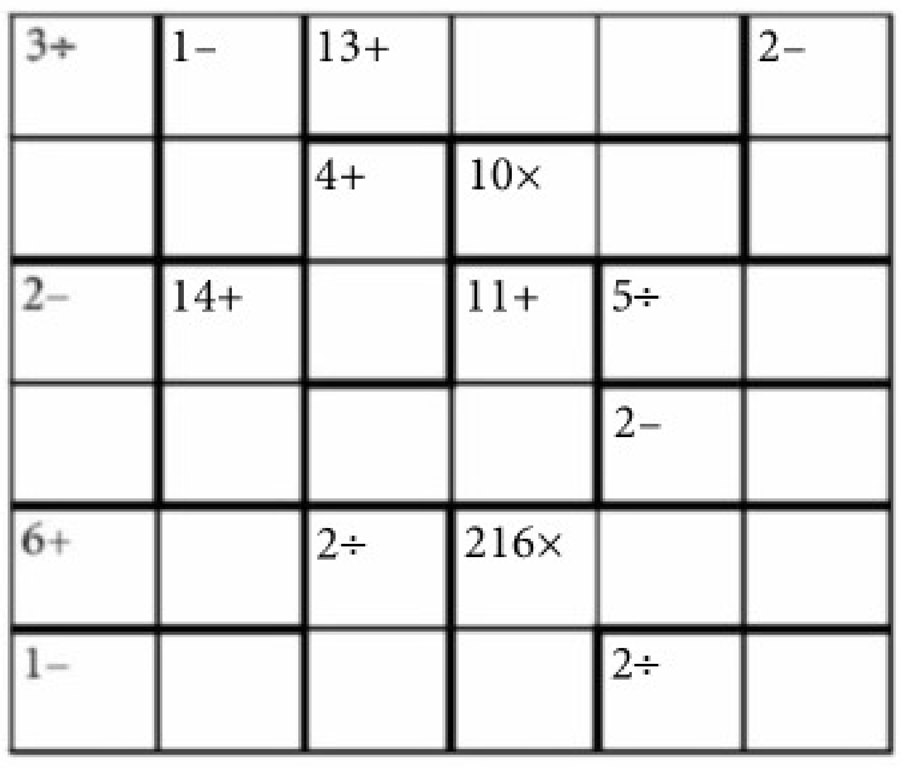
PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



3+	1–	13+			2–
		4+	10×		
2–	14+		11+	5÷	
				2–	
6+		2÷	216×		
1–				2÷	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

OLYMPIAD CORNER

1. Find the equation of the locus of a point P which moves so that the tangents from P to the circle $x^2 + y^2 = r^2$ cut off a line segment of length $2r$ on the line $x = r$.

2. Suppose than $n \geq 3$. Let $A_1, \ldots, A_n$ be a convex n-gon (as usual with interior angles $A_1, \ldots, A_n$).
Determine the greatest constant C_n such that
$$\sum_{k=1}^{n} \frac{1}{A_k} \geq C_n \sum_{k=1}^{n} \frac{1}{\pi - A_k}.$$
Determine when equality occurs.

3. Suppose that the function $f(u)$ has a second derivative in the interval (a, b), and that $f(u) \geq 0$ for all $u \in (a, b)$. Prove that
   1. $(y - z) f(x) + (z - x) f(y) + (x - y) f(z) > 0$ for all $x, y, z \in (a, b)$, $z < y < x$ if and only if $f''(u) > 0$ for all $u \in (a, b)$.
   2. $(y - z) f(x) + (z - x) f(y) + (x - y) f(z) = 0$ for all $x, y, z \in (a, b)$, $z < y < x$ if and only if $f(u)$ is a linear function on (a, b).

4. Prove that for $a_1, a_2, \ldots, a_n > 0$,
$$\frac{(a_1 + a_2 + \ldots + a_n)^2}{2(a_1^2 + a_2^2 + \ldots + a_n^2)} \leq \frac{a_1}{a_2 + a_3} + \frac{a_2}{a_3 + a_4} + \ldots + \frac{a_n}{a_1 + a_2}.$$

5. $a_1, \ldots, a_k, a_{k+1}, \ldots, a_n$ are positive numbers $(k < n)$. Suppose that the values of $a_{k+1}, \ldots, a_n$ are fixed. How should one choose the values of $a_1, \ldots, a_n$ in order to minimize $\sum_{i, j, i \neq j} \frac{a_i}{a_j}$?

6. Compute tan 20° tan 40° tan 60° tan 80°.

7. Let a_1, a_2, a_3, a_4, a_5, be a five-term geometric sequence satisfying the inequality $0 < a_1 < a_2 < a_3 < a_4 < a_5 < 100$, where each term is an integer. How many of these five-term geometric sequences are there?

SOLUTIONS

1. From the circle, $x^2 + y^2 = a^2$, we find the points of intersection with the line $(y - q) = m(x - p)$.
Substituting $y = mx - mp + q$, we get
$x^2 + (mx - mp + q)^2 = a^2$, leading to the two solutions :
$$x = \frac{m(mp - q) \pm \sqrt{a^2(m^2 + 1) - m^2 p^2 + q(2mp - q)}}{m^2 + 1}$$
For the line to be a tangent, the discriminant, $\Delta = a^2(m^2 + 1) - m^2p^2 + q(2mp - q)$, must be zero. So we solve $\Delta = 0$, to get
$$m = \frac{pq}{p^2 - a^2} \pm \frac{a\sqrt{p^2 + q^2 - a^2}}{p^2 - a^2}.$$
So, we have that the equations of the tangent lines from (p, q) to the circle $x^2 + y^2 = a^2$ are :
$$y = \left(-\frac{pq}{a^2 - p^2} \pm \frac{a\sqrt{a^2 - p^2 - q^2}}{p^2 - a^2} \right) x$$
$$- \left(-pq/(a^2 - p^2) \pm \frac{\sqrt{a^2 - p^2 - q^2}}{p^2 - a^2} \right) p + q.$$
Setting $x = a$ in these, and subtracting the two values leads to
$(a + p)^2 = p^2 + q^2 - a^2$.
Thus the locus of P is given by
$y^2 = 2a(a + x)$.
This is a parabola opening to the right with central axis, the x-axis and nose at $x = -a$. There are, ofcourse, three points that should be excluded : $(-a, 0)$, $(a, 2a)$ and $(a, -2a)$.

2. We assume that the intention of the problem is to determine the largest constant C_n so that the given inequality holds for all convex n-gons.
If $n \geq 4$ then clearly $C_n = 0$, since we could let one of the interior angles tend to π without forcing any other interior angle to tend to zero.

Then $\sum_{k=1}^{n}\frac{1}{\pi-A_k}\to+\infty$ while $\sum_{k=1}^{n}\frac{1}{A_k}$ is bounded.

Thus the inequality cannot hold unless $C_n \le 0$. Hence $C_n = 0$ and equality can never occur.

If $n = 3$, then by the arithmetic-harmonic-mean inequality.

$$(A_1+A_2)\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\ge 4$$

and thus $\frac{1}{A_1}+\frac{1}{A_2}\ge\frac{4}{A_1+A_2}=\frac{4}{\pi-A_3}$

Similarly, $\frac{1}{A_2}+\frac{1}{A_3}\ge\frac{4}{\pi-A_1}$ and $\frac{1}{A_3}+\frac{1}{A_1}\ge\frac{4}{\pi-A_2}$

Adding, we get

$$\sum_{k=1}^{3}\frac{1}{A_k}\ge 2\sum_{k=1}^{3}\frac{1}{\pi-A_k}$$

It is easily seen that equality holds if and only if all the A_k's are equal. Hence $C_3 = 2$ and equality holds if and only if $\Delta A_1A_2A_3$ is equilateral.

3. Part 1 is incorrect. The first statement is actually equivalent to "f is strictly convex", not "$f''(u) > 0$ for all $u \in (a, b)$". Strictly convex means that $f(tx + (1 - t)y) < t f(x) + (1 - t) f(y)$ for $x, y \in (a, b)$ and $0 < t < 1$. If f is twice differentiable this is equivalent to $f''(u) \ge 0$ everywhere with $f''(u) > 0$ on a dense set. For example, $f(x) = x^4$ is strictly convex everywhere but $f''(0) = 0$, and in this case

$$(y - z)x^4 + (z - x)y^4 + (x - y)z^4$$

$$=(x-z)(y-z)(x-y)\left(\frac{(x+y)^2+(y+z)^2+(x+z)^2}{2}\right)>0$$

if $z < y < x$.

More generally, let f be any continuous function on (a, b) and $n \ge 3$ an integer. Given $x_1, x_2, \ldots, x_n$ let

$$L(f)=\sum_{i=1}^{n}(x_{i-1}-x_{i+1})f(x_i),$$

where we take $x_0 = x_n$ and $x_{n+1} = x_1$. Then the following are equivalent :

(i) $L(f) > 0$ whenever $a < x_1 < x_2 < \ldots < x_n < b$.

(ii) f is strictly convex on (a, b).

Similarly, for part 2, we will show that f is linear on (a, b) if and only if $L(f) = 0$ whenever $a < x_1 < x_2 < \ldots < x_n < b$.

Proof of (i) $\Rightarrow$ (ii) : If f is not strictly convex, then either it is not convex at all or its graph includes a straight line segment. In the first case, there exist x, z and t with $a < z < x < b$ and $0 < t < 1$ such that $f(tz + (1 - t)x) > tf(z) + (1 - t) f(x)$. Taking $y = tz + (1 - t)x$ and nothing that $t = (x - y)/(x - z)$ we can rewrite this as

$$f(y)>\frac{x-y}{x-z}f(z)+\frac{y-z}{x-z}f(x)$$

or $(y - z) f(x) + (z - x) f(y) + (x - y) f(z) < 0$. Let $x_1 = z$, $x_2 = y$ and $x_3 = \ldots = x_n = x$, and we have $L(f) < 0$ (if $n > 3$, note that the terms in $f(x_j)$ for $3 < j < n$ are 0 and the terms in $f(x_3)$ and $f(x_n)$ add to $(y - x) f(x)$). If $n > 3$ we must move $x_3, \ldots, x_n$ slightly so that $x_3 < \ldots < x_n$, but by continuity the inequality will still be true if the changes are small enough. In the second case, we can take any $x_1 < \ldots < x_n$ in the interval over which the graph is a straight line $f(x) = cx + d$, and then

$$L(f)=c\left(\sum_{i=1}^{n}x_{i-1}x_i-\sum_{i=1}^{n}x_ix_{i+1}\right)+d\left(\sum_{i=1}^{n}x_{i-1}-\sum_{i=1}^{n}x_{i+1}\right)=0$$

Proof of (ii) $\Rightarrow$ (i) : As noted above, if g is linear on (a, b) then $L(g) = 0$. Thus we can add a linear function g to f without changing either (i) or (ii).

With an appropriate choice of g we can make $f(x_1) = f(x_n) = 0$, and by strict convexity we then have $f(x_j) < 0$ for $1 < j < n$. Now write

$$L(f)=\sum_{i=1}^{n}(x_{i-1}-x_i)f(x_i)+\sum_{i=1}^{n}(x_i-x_{i+1})f(x_i)$$

$$=\sum_{i=1}^{n}(x_{i-1}-x_i)(f(x_i)+f(x_{i-1}))$$

Note that the $i = 1$ term is 0, and each of the other terms is strictly positive, so the sum is strictly positive.

As noted above, if f is linear on (a, b) then $L(f) = 0$. Conversely, suppose $L(f) = 0$ whenever $a < x_1 < x_2 < \ldots < x_n < b$. By part 1, $L(h) > 0$ where $h(x) = x^2$, and so since L is a linear operator $L(f + ch) = cL(h) > 0$ for any $c > 0$. Again using part 1, $f + ch$ is strictly convex, and so $f=\lim\limits_{c\to 0^+}(f+ch)$ is convex on (a, b). By the same argument, $-f$ is convex, so f is concave on (a, b). Any function that is both convex and concave on (a, b) is linear on (a, b).

4. Recall that CSB states that for all real x_i, y_i, $i = 1, 2, \ldots, n$, $(x_1y_1 + x_2y_2 +\ldots. + x_ny_n)^2 \le (x_1^2 + x_2^2 + \cdots + x_n^2)(y_1^2 + y_2^2 + \cdots + y_n^2)$.

Setting $x_i=\sqrt[4]{a_i},\ y_i=\sqrt[4]{a_i^3}$, we obtain

$(a_1 + a_2 + \cdots + a_n)^2$

$\le(\sqrt{a_1}+\sqrt{a_2}+\cdots+\sqrt{a_n})(a_1\sqrt{a_1}+a_2\sqrt{a_2}+\cdots+a_n\sqrt{a_n})$

Setting $x_i=\sqrt{a_i},\ y_i=a_i$, we obtain

$$(a_1\sqrt{a_1}+a_2\sqrt{a_2}+\cdots+a_n\sqrt{a_n})^2$$
$$\le (a_1+a_2+\cdots+a_n)(a_1^2+a_2^2+\cdots+a_n^2)$$

Combining these two, we obtain

$$(a_1+a_2+\cdots+a_n)^3$$
$$\le (\sqrt{a_1}+\sqrt{a_2}+\ldots+\sqrt{a_n})^2(a_1^2+a_2^2+\ldots+a_n^2)$$

Finally, setting $x_i=\sqrt{a_{i+1}+a_{i+2}}$, $y_i=\sqrt{\frac{a_i}{a_{i+1}+a_{i+2}}}$, we obtain

$$(\sqrt{a_1}+\sqrt{a_2}+\ldots+\sqrt{a_n})^2$$
$$\le 2(a_1+a_2+\cdots+a_n)\left(\frac{a_1}{a_2+a_3}+\frac{a_2}{a_3+a_4}+\cdots+\frac{a_n}{a_1+a_2}\right)$$

The last two inequalities give the desired result.

5. To minimize the given rational function, choose

$$a_i=\left(\frac{a_{k+1}+\ldots+a_n}{\frac{1}{a_{k+1}}+\ldots+\frac{1}{a_n}}\right)^{1/2}=(\mathrm{A}\cdot\mathrm{H})^{1/2}, i=1,2,\ldots,k$$

where A is the arithmetic and H the harmonic mean of $a_{k+1}, \ldots, a_n$.

To prove this, we will be forgiven if we change notation : let $x_i = a_i$, $i = 1, 2, \ldots, k$ and $b_r = a_{k+r}$, $r = 1, \ldots, m$ with $k + m = n$, and denote the given rational function $F(x_1, \ldots, x_k)$. Then we have $F(x_1, \ldots, x_k) = X + Y + B$, where

$$X=\sum_{1\le i<j\le k}\left(\frac{x_i}{x_j}+\frac{x_j}{x_i}\right),\quad Y=\sum_{1\le i\le k}\ \sum_{1\le r\le m}\left(\frac{x_i}{b_r}+\frac{b_r}{x_i}\right),$$

$$B=\sum_{1\le r<s\le m}\left(\frac{b_r}{b_s}+\frac{b_s}{b_r}\right)$$

Note that B is fixed and Y can be improved to

$$Y=\sum_{1\le i\le k}\left(\left(\sum_{1\le r\le m}\frac{1}{b_r}\right)x_i+\left(\sum_{1\le r\le m}b_i\right)\frac{1}{x_i}\right)$$

$$=\sum_i\left(\frac{m}{\mathrm{H}}x_i+\frac{m\mathrm{A}}{x_i}\right)$$

where A is the arithmetic mean and H is the harmonic mean of the b_r.

Now we recall that the simple function $\alpha x+\frac{\beta}{x}$ (with α, β, x all positive) assumes its minimum when $\alpha x=\frac{\beta}{x}$; that is $x=\sqrt{\beta/\alpha}$. Thus each of the terms in Y (and so Y itself) assumes its minimum when we choose, for $i = 1, 2, \ldots, k$,

$$x_i=\sqrt{\frac{m\mathrm{A}}{(m/\mathrm{H})}}=\sqrt{\mathrm{AH}},\text{ as asserted.}$$

But there is more. It is also known that each term in X, (and so X itself) assumes its minimum when $x_i = x_j$, with $1 \le i < j \le k$. Thus choosing all $x_i=\sqrt{\mathrm{AH}}$ minimizes both X and Y and, since B is fixed, minimizes $F(x_1, \ldots, x_k)$ as claimed.

6. Let $A = \tan 20° \tan 40° \tan 60° \tan 80°$. Then $A=\frac{B}{C}$, where $B = 8 \sin 20° \sin 40° \sin 60° \sin 80°$ and $C = 8 \cos 20° \cos 40° \cos 60° \cos 80°$. Then

$$B = 8 \sin 20° \sin 40° \sin 60° \sin 80°$$
$$= 2(\cos 60° + \cos 100°)(\cos 20° - \cos 100°)$$
$$= (1 + 2 \sin 10°)(\cos 20° + \sin 10°)$$
$$= \cos 20° + \sin 10° + 2 \sin 10° \cos 20° + 2 \sin^2 10°$$
$$= \cos 20° + \sin 10° + \sin 30° - \sin 10° + (1 - \cos 20°)$$
$$= \frac{3}{2},$$

$$C = 8 \cos 20° \cos 40° \cos 60° \cos 80°$$
$$= 2(\cos 60° + \cos 100°)(\cos 20° + \cos 100°)$$
$$= (1 - 2 \sin 10°)(\cos 20° - \sin 10°)$$
$$= \cos 20° - \sin 10° - 2 \sin 10° \cos 20° + 2 \sin^2 10°$$
$$= \cos 20° - \sin 10° - \sin 30° + \sin 10° + (1 - \cos 20°)$$
$$= \frac{1}{2}.$$

Therefore, $A=\frac{B}{C}=3.$

7. Let $\frac{n}{m}$ be the common ratio of the geometric sequence, where n and m are relatively prime integers, with $n > m$. Now $a_5=a_1\times\frac{n^4}{m^4}$, so let $a_1 = km^4$, where k is a positive integer. Thus, our geometric series becomes km^4, km^3n, km^2n^2, kmn^3, kn^4 and $kn^4 < 100$. If $n > 4$, then $kn^4 \ge n^4 > 256 > 100$. So $n \le 3$. Hence, there are three cases to consider :

If $n = 3$ and $m = 2$, then $81k < 100$, so $k = 1$. The only solution is (16, 24, 36, 54, 81).

If $n = 3$ and $m = 1$, then $81k < 100$, so $k = 1$. The only solution is (1, 3, 9, 27, 81).

If $n = 2$ and $m = 1$, then $16k < 100$, so $k = 1, 2, \ldots, 6$. There are six solutions, namely (1, 2, 4, 8, 16), (2, 4, 8, 16, 32), (3, 6, 12, 24, 48), (4, 8, 16, 32, 64), (5, 10, 20, 40, 80) and (6, 12, 24, 48, 96). In total, there are eight sequences.

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YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. If $\exp[(\sin^2 x + \sin^4 x + \sin^6 x + \ldots\ldots\ldots\ldots \infty)\ln 2]$ satisfies the equation $x^2 - 9x + 8 = 0$, then find the value of $\dfrac{\cos x}{\cos x + \sin x}, 0 < x < \dfrac{\pi}{2}$. *(Ekansh Goyal, U.P.)*

Ans. Let $S = \sin^2 x + \sin^4 x + \sin^6 x + \ldots\ldots\ldots\ldots \infty$

$$= \frac{\sin^2 x}{1 - \sin^2 x} = \tan^2 x$$

Thus, $e^{S \ln 2} = e^{\ln 2^S} = 2^S = 2^{\tan^2 x}$

and roots of the equation $x^2 - 9x + 8 = 0$ are $x = 1, 8$.

According to the given condition, we have

$2^{\tan^2 x} = 1, 8$ *i.e.*, $2^0, 2^3$

i.e., $\tan^2 x = 0, 3 \Rightarrow \tan x = 0, \pm\sqrt{3}$

$$\therefore \quad \tan x = \sqrt{3} \qquad \left[\because x \text{ lies in } \left(0, \frac{\pi}{2}\right)\right]$$

Hence, $\dfrac{\cos x}{\cos x + \sin x} = \dfrac{1}{1 + \tan x} = \dfrac{1}{1 + \sqrt{3}}$

2. If $a < b < c < d$, then prove that for any real λ, the quadratic equation $(x - a)(x - c) + \lambda(x - b)(x - d) = 0$ has real roots. *(Aman Bansal, Bihar)*

Ans. The given equation reduces to

$$(1 + \lambda)x^2 - \{(a + c) + \lambda(b + d)\}x + (ac + \lambda bd) = 0 \quad \ldots(i)$$

Now, if the roots are real, then discriminant ≥ 0

i.e., $\{a + c + \lambda(b + d)\}^2 - 4(1 + \lambda)(ac + \lambda bd) \geq 0$

i.e., $\{(b + d)^2 - 4bd\}\lambda^2 + \{2(a + c)(b + d) - 4(ac + bd)\}\lambda + \{(a + c)^2 - 4ac\} \geq 0$

i.e., $(b - d)^2\lambda^2 + 2\{(a + c)(b + d) - 2(ac + bd)\}\lambda + (a - c)^2 \geq 0 \quad \ldots(ii)$

The discriminant of quadratic expression (ii) is

$$D = 4[(a + c)(b + d) - 2(ac + bd)]^2 - 4(b - d)^2(a - c)^2$$
$$= 4[ab + bc + ad + cd - 2ac - 2bd + (b - d)(a - c)] \times [ab + bc + ad + cd - 2ac - 2bd - (b - d)(a - c)]$$
$$[\because x^2 - y^2 = (x + y)(x - y)]$$
$$= 4[2ab + 2cd - 2ac - 2bd][2bc + 2ad - 2ac - 2bd]$$
$$= 16[ab + cd - ac - bd][bc + ad - ac - bd]$$
$$= 16(a - b)(b - c)(c - d)(d - a) < 0 \qquad [\because a < b < c < d]$$

Also, coeff. of $\lambda^2 = (b - d)^2$, which is +ve.

Hence, inequality (ii) is true for any real λ which proves that equation (i) has real roots for any real λ.

3. Four numbers are randomly chosen from the first hundred natural numbers. Find the probability that the numbers are in arithmetic progression.

(Animesh Bohra, West Bengal)

Ans. Let sample space S = {choosing 4 numbers out of 100 numbers}

$\therefore \quad n(S) = {}^{100}C_4$

Let A_1 = {four numbers having common difference 1}
$= \{(1, 2, 3, 4), (2, 3, 4, 5), (3, 4, 5, 6), \ldots\ldots, (97, 98, 99, 100)\}$

Then $n(A_1) = 97$

Let A_2 = {four numbers having common difference 2}
$= \{(1, 3, 5, 7), (2, 4, 6, 8), (3, 5, 7, 9), \ldots\ldots, (94, 96, 98, 100)\}$

Then $n(A_2) = 94$

Proceeding similarly, we can see that four numbers can be chosen with a maximum common difference of 33, viz. (1, 34, 67, 100). Then $n(A_{33}) = 1$

Thus the number of favourable ways is

$$n(A_1) + n(A_2) + \ldots. + n(A_{33}) = 97 + 94 + 91 + \ldots. + 1$$
$$= 33\left(\frac{98}{2}\right) = 33 \cdot 49$$

Hence, the required probability is $\dfrac{33 \cdot 49}{{}^{100}C_4}$

$$= \frac{33 \cdot 49 \cdot 24}{100 \cdot 99 \cdot 98 \cdot 97} = \frac{1}{25 \cdot 97}.$$

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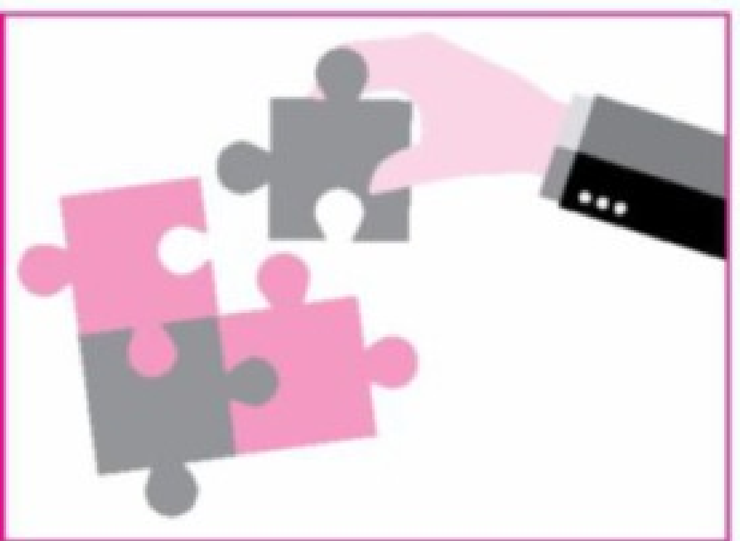
PUZZLE CORNER



ANSWER · SEPTEMBER 2019

3+ 1	7+ 4	3	60× 2	6	5
2	6+ 1	20× 4	5	15× 3	6× 6
3	2	2− 6	4	5	1
2− 4	6	15× 5	3	1	5+ 2
1− 6	10× 5	2	24× 1	4	3
5	3÷ 3	1	6	2− 2	4

Between
6th to 11th
January

FOCUS JEE MAIN 2020 PRACTICE PAPER

with Numerical Value Type Questions

Single Option Correct Type

1. The domain of definition of the function $y = \dfrac{1}{\log_{10}(1-x)} + \sqrt{x+2}$ is

(a) $(-3, -2)$ excluding -2.5 (b) $[0, 1]$ excluding 0.5
(c) $[-2, 1)$ excluding 0 (d) none of these

2. If z is a non-real complex number, then the minimum value of $\text{Im}\, z^5/(\text{Im} z)^5$ is

(a) -1 (b) -2 (c) -4 (d) -5

3. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $Au_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, then $u_1 + u_2$ is equal to

(a) $\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

4. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\displaystyle\lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(a) 0 (b) $\dfrac{a^2}{2}(\alpha-\beta)^2$

(c) $\dfrac{1}{2}(\alpha-\beta)^2$ (d) $\dfrac{-a^2}{2}(\alpha-\beta)^2$

5. If $\alpha + \beta + \gamma = 2\pi$ then

(a) $\tan\frac{1}{2}\alpha + \tan\frac{1}{2}\beta + \tan\frac{1}{2}\gamma = \tan\frac{1}{2}\alpha \tan\frac{1}{2}\beta \tan\frac{1}{2}\gamma$

(b) $\tan\frac{1}{2}\alpha \tan\frac{1}{2}\beta + \tan\frac{1}{2}\beta \tan\frac{1}{2}\gamma + \tan\frac{1}{2}\gamma \tan\frac{1}{2}\alpha = 1$

(c) $\tan\frac{1}{2}\alpha + \tan\frac{1}{2}\beta + \tan\frac{1}{2}\gamma = -\tan\frac{1}{2}\alpha \tan\frac{1}{2}\beta \tan\frac{1}{2}\gamma$

(d) none of these

6. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of $\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \left(z^3+\frac{1}{z^3}\right)^2 + \ldots + \left(z^6+\frac{1}{z^6}\right)^2$ is

(a) 18 (b) 54 (c) 6 (d) 12

7. The value of $\displaystyle\int_0^1 \frac{8\ln(1+x)}{1+x^2}\,dx$ is

(a) $\dfrac{\pi}{2}\ln 2$ (b) $\ln 2$

(c) $\pi \ln 2$ (d) $\dfrac{\pi}{8}\ln 2$

8. Solution of the differential equation $\cos x\,dy = y(\sin x - y)dx$, $0 < x < \pi/2$ is

(a) $\sec x = (\tan x + c)y$ (b) $y \sec x = \tan x + c$
(c) $y \tan x = \sec x + c$ (d) $\tan x = (\sec x + c)y$

9. If 1, $\log_9(3^{1-x} + 2)$, $\log_3[4 \cdot 3^x - 1]$ are in A.P. then x equals

(a) $\log_3 4$ (b) $1 - \log_3 4$
(c) $1 - \log_4 3$ (d) $\log_4 3$

10. Let $f : R \to R$ be any function. Define $g : R \to R$ by $g(x) = |f(x)|$ for all x. Then g is
(a) onto if f is onto
(b) one-one if f is one-one
(c) continuous if f is continuous
(d) differentiable if f is differentiable

11. If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$, then a_n is
(a) $\frac{b^n - a^n}{b-a}$ (b) $\frac{a^n - b^n}{b-a}$
(c) $\frac{a^{n+1} - b^{n+1}}{b-a}$ (d) $\frac{b^{n+1} - a^{n+1}}{b-a}$

12. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta\sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is
(a) $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$ (b) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$
(c) $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$ (d) $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$

13. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals
(a) 3 (b) -1 (c) 1 (d) -3

14. PQ is a diameter of a circle and R is any point on the circumference of the circle. Then
(a) the area of ΔPQR is maximum when it is isosceles
(b) the area of ΔPQR is minimum when it is isosceles
(c) the perimeter of ΔPQR is minimum when it is isosceles
(d) none of these

15. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is
(a) $4x^2 + y^2 = 8$ (b) $x^2 + 4y^2 = 16$
(c) $4x^2 + y^2 = 4$ (d) $x^2 + 4y^2 = 8$

16. The line passing through the points $(5, 1, a)$ and $(3, b, 1)$ crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then
(a) $a = 8, b = 2$ (b) $a = 2, b = 8$
(c) $a = 4, b = 6$ (d) $a = 6, b = 4$

17. Suppose a population A has 100 observations 101, 102, ..., 200, and another population B has 100 observations 151, 152, ..., 250. If V_A and V_B represent the variances of the two populations, respectively, then V_A / V_B is
(a) 1 (b) 9/4
(c) 4/9 (d) 2/3

18. Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}| = 1, |\vec{v}| = 2, |\vec{w}| = 3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}, \vec{w}$ are perpendicular to each other then $|\vec{u} - \vec{v} + \vec{w}|$ equals
(a) $\sqrt{14}$ (b) $\sqrt{7}$ (c) 2 (d) 14

19. **Statement-1 :** $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.
Statement-2 : $\sim (p \leftrightarrow \sim q)$ is a tautology.
(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(b) Statement- 1 is true, Statement-2 is false
(c) Statement-1 is false, Statement-2 is true
(d) Statement-1 is true, Statement-2 is true; Statement-2 is correct explanation for Statement-1

20. The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is
(a) $\frac{47}{4}$ units (b) 13 units
(c) 39 units (d) 26 units

Numerical Answer Type

21. Let ω be the complex number. Then the number of distinct complex numbers z satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is

22. A five digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways this can be done is

23. If $\cos\alpha + \cos\beta = \frac{3}{2}$ and $\sin\alpha + \sin\beta = \frac{1}{2}$ and θ is the arithmetic mean of α and β, then $\sin 2\theta + \cos 2\theta$ is equal to

24. Fifth term of a GP is 2, then the product of its 9 terms is

25. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is

SOLUTIONS

1. **(c) :** Let $y = f(x) + g(x)$ where

$$f(x) = \frac{1}{\log_{10}(1-x)}, g(x) = \sqrt{x+2}$$

The domain of the given function is $D_f \cap D_g$

$f(x)$ will be defined if $1 - x > 0$ and $x \neq 0$

$\Rightarrow \quad x < 1$ and $x \neq 0$

$\therefore \quad D_f = (-\infty, 1) - \{0\}$

For domain of $g(x) = \sqrt{x+2}$, $x + 2 \geq 0 \Rightarrow x \geq -2$

$\therefore \quad D_g = [-2, \infty)$

Common domain $= [-2, 1) - \{0\}$

2. **(c) :** Let $z = re^{i\theta}$

$$\therefore \quad \frac{\operatorname{Im} z^5}{(\operatorname{Im} z)^5} = \frac{r^5(\sin 5\theta)}{r^5(\sin\theta)^5} = \frac{\sin 5\theta}{\sin^5\theta}.$$

Now, $\dfrac{d}{d\theta}\left(\dfrac{\operatorname{Im} z^5}{(\operatorname{Im} z)^5}\right)$

$$= \frac{\sin^5\theta \cdot 5\cos 5\theta - 5\sin 5\theta \sin^4\theta \cos\theta}{(\sin^5\theta)^2}$$

Put $\dfrac{d}{d\theta}\left(\dfrac{\operatorname{Im} z^5}{(\operatorname{Im} z)^5}\right) = 0$

$\Rightarrow \quad 5\sin^4\theta\,(\sin\theta\cos 5\theta - \cos\theta\sin 5\theta) = 0$

$\Rightarrow \quad \sin\theta = 0$ or $\sin(-4\theta) = 0$

$\Rightarrow \quad \theta = n\pi$ or $\theta = \dfrac{n\pi}{4}$, where $n \in Z$

As z is non–real complex number.

$\therefore \quad$ only $\theta = \dfrac{n\pi}{4}$ is possible

$$\theta = -\frac{\pi}{4} \Rightarrow \min\left[\frac{\operatorname{Im} z^5}{(\operatorname{Im} z)^5}\right] = -4$$

3. **(b) :** Given, $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

Let $u_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $\therefore Au_1 = \begin{bmatrix} a \\ 2a+b \\ 3a+2b+c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ (Given)

Comparing both the matrices, we get

$a = 1, 2a + b = 0, 3a + 2b + c = 0$

$\Rightarrow \quad b = -2, c = 1$

Let $u_2 = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ $\therefore Au_2 = \begin{bmatrix} p \\ 2p+q \\ 3p+2q+r \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ (Given)

Comparing both the matrices, we get

$p = 0, 2p + q = 1, 3p + 2q + r = 0 \Rightarrow q = 1, r = -2$

$$\therefore \quad u_1 + u_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

4. **(b) :** As α is a root of $ax^2 + bx + c = 0$

$\therefore \; a\alpha^2 + b\alpha + c = 0.$

Now, $\displaystyle\lim_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x-\alpha)^2}$

$$= \lim_{x \to \alpha} \frac{2\sin^2\left(\dfrac{ax^2 + bx + c}{2}\right)}{(x-\alpha)^2}$$

$$= \lim_{x \to \alpha} \frac{2\sin^2\left[\dfrac{a(x-\alpha)(x-\beta)}{2}\right]}{a^2\left[\dfrac{(x-\alpha)^2(x-\beta)^2}{4}\right]} \times \frac{a^2(x-\beta)^2}{4}$$

$$= \lim_{x \to \alpha} \left[\frac{\sin\left(\dfrac{a(x-\alpha)(x-\beta)}{2}\right)}{\dfrac{a(x-\alpha)(x-\beta)}{2}}\right]^2 \times \frac{a^2(x-\beta)^2}{2}$$

$$= 1 \times \frac{a^2}{2}(\alpha-\beta)^2 = \frac{a^2}{2}(\alpha-\beta)^2$$

5. **(a) :** We have, $\dfrac{\alpha}{2} + \dfrac{\beta}{2} + \dfrac{\gamma}{2} = \pi \Rightarrow \dfrac{\alpha}{2} + \dfrac{\beta}{2} = \pi - \dfrac{\gamma}{2}$

$$\therefore \quad \tan\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \tan\left(\pi - \frac{\gamma}{2}\right) = -\tan\frac{\gamma}{2}$$

$$\Rightarrow \quad \frac{\tan\dfrac{\alpha}{2} + \tan\dfrac{\beta}{2}}{1 - \tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2}} = -\frac{\tan\dfrac{\gamma}{2}}{1}$$

$$\therefore \quad \tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$$

6. **(d) :** We have, $z^2 + z + 1 = 0$

$\Rightarrow \quad z = \omega, \omega^2$

$$\therefore \quad \left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \ldots\ldots + \left(z^6 + \frac{1}{z^6}\right)^2$$

$= 4\,(\omega + \omega^2)^2 + 2(\omega^3 + \omega^3)^2 = 4\,(-1)^2 + 2(2^2)$

$= 4 + 8 = 12$

7. **(c) :** Let $I = \displaystyle\int_0^1 \frac{8\ln(1+x)}{1+x^2}dx$

Let $J = \displaystyle\int_0^1 \frac{\ln(1+x)}{1+x^2}dx$

Let $x = \tan\theta \Rightarrow J = \displaystyle\int_0^{\pi/4} \ln(1+\tan\theta)d\theta$

Now $J = \int_0^{\pi/4} \ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$

Adding $2J = \int_0^{\pi/4} \left[\ln(1+\tan\theta)+\ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)\right]d\theta$

$\Rightarrow \quad 2J = \int_0^{\pi/4} \ln\left\{(1+\tan\theta)\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)\right\}d\theta$

$\Rightarrow \quad 2J = \int_0^{\pi/4} (\ln 2)d\theta = \frac{\pi}{4}\ln 2 \Rightarrow 8J = \pi\ln 2$

$\Rightarrow \quad I = 8J = \pi\ln 2$

8. (a) : $\frac{dy}{dx} = \frac{y(\sin x - y)}{\cos x} \Rightarrow \frac{dy}{dx} = y\tan x - y^2\sec x$

$\Rightarrow \quad \frac{dy}{dx} - y\tan x = -y^2\sec x$

$\Rightarrow \quad \frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x$

Putting, $-\frac{1}{y} = v$, we have

$\frac{dv}{dx} + (\tan x)v = -\sec x$, which is linear in v.

$\therefore \quad \text{I.F.} = e^{\int \tan x\, dx} = e^{\ln\sec x} = \sec x$

The solution is $v \times \sec x = \int -\sec^2 x\, dx + k$

$\Rightarrow \quad v\sec x = -\tan x + k$

$\Rightarrow \quad -\frac{\sec x}{y} = -\tan x - c \Rightarrow \sec x = y(\tan x + c)$

9. (b) : As $1, \frac{1}{2}\log_3(3^{1-x}+2), \log_3(4\cdot 3^x - 1) \in$ A.P.

$\Rightarrow \quad \log_3(3^{1-x}+2) = \log_3(4\cdot 3^x - 1) + 1$

$\Rightarrow \quad 3^{1-x} + 2 = (4\cdot 3^x - 1) \times 3$

$\Rightarrow \quad 3^{1-x} + 2 = 12\cdot 3^x - 3$

$\Rightarrow \quad 3^x[(3^{1-x}) + 2] = 12\cdot 3^{2x} - 3\cdot 3^x$

(Multiplying 3^x on both sides)

$\Rightarrow \quad 12t^2 - 5t - 3 = 0$, where $t = 3^x$

$\Rightarrow \quad (3t+1)(4t-3) = 0 \Rightarrow t = -1/3, t = 3/4$

$\Rightarrow \quad 3^x = -1/3$ which is not possible so $3^x = \frac{3}{4}$

$\Rightarrow \quad x\log_3 3 = \log_3 3 - \log_3 4$

(By taking logarithm at the base 3 both sides)

$\Rightarrow \quad x = 1 - \log_3 4$

10. (c) : f is continuous at $x = a$, if

$$\lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(a+h) = f(a)$$

Now, $g(x) = |f(x)|$

$g(a-0) = \lim_{h\to 0} g(a-h) = \lim_{h\to 0}|f(a-h)| = |f(a)|$

Similarly, $g(a+0) = \lim_{h\to 0} g(a+h)$

$= \lim_{h\to 0}|f(a+h)| = |f(a)| = g(a)$

$\Rightarrow \quad g$ is continuous at $x = a$.

Hence g is continuous if f is continuous.

11. (d) : Given, $(a_0 + a_1x + \ldots + a_nx^n + \ldots)$

$= (1-ax)^{-1}(1-bx)^{-1}$

$= (1 + ax + a^2x^2 + \ldots + a^{n-1}x^{n-1} + a^nx^n + \ldots)$

$(1 + bx + b^2x^2 + \ldots + b^nx^n + \ldots)$

$\Rightarrow \quad (a_0 + a_1x + \ldots + a_nx^n + \ldots)$

$= 1 + x(a+b) + x^2(a^2+ab+b^2) + x^3(a^3 + a^2b + ab^2 + b^3) + \ldots + x^n(a^n + a^{n-1}b + a^{n-2}b^2 + \ldots + ab^{n-1} + b^n) + \ldots$

On comparing the coefficient of x^n both sides, we have

$a^n = a^n + a^{n-1}b + a^{n-2}b^2 + \ldots + a\,b^{n-1} + b^n$

$= \frac{(a^n + a^{n-1}b + a^{n-2}b^2 + \ldots + ab^{n-1} + b^n)(b-a)}{b-a}$

[Multiplying and dividing by $(b-a)$]

$= \frac{b^{n+1} - a^{n+1}}{b-a}$

12. (c) : According to question

$\int_{\pi/4}^{\beta} f(x)dx = \left(\beta\sin\beta + \frac{\pi}{4}\cos\beta + \beta\sqrt{2}\right)$

$f(\beta) = \sin\beta + \beta\cos\beta - \frac{\pi}{4}\sin\beta + \sqrt{2}$

$\therefore \quad f\left(\frac{\pi}{2}\right) = 1 - \frac{\pi}{4} + \sqrt{2}$

13. (d) : The equation $ax^2 + 2hxy + by^2 = 0$

$= (y - m_1x)(y - m_2x)$

$\Rightarrow \quad m_1 + m_2 = -\frac{2h}{b} = \frac{1}{4c} \quad \ldots(*)$

$m_1m_2 = \frac{3}{2}c$

and $3x + 4y = 0 \Rightarrow m_1 = -3/4 \;\therefore\; m_2 = -\frac{2}{c}$

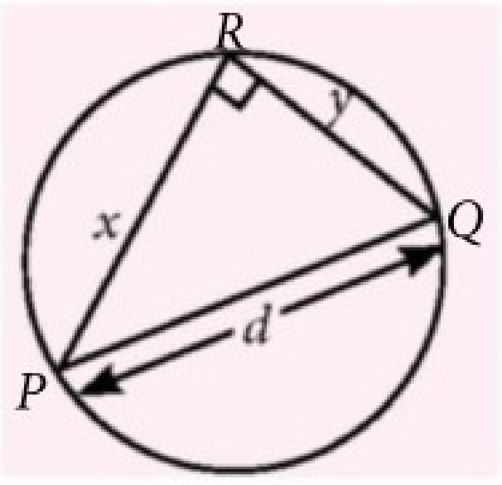
Now by (*) we have

$-\left(\frac{3}{4} + \frac{2}{c}\right) = \frac{1}{4c}$

$\Rightarrow \quad -\frac{3}{4} = \frac{1}{4c} + \frac{2}{c} \Rightarrow -\frac{3}{4} = \frac{1}{4c} + \frac{8}{4c}$

$\Rightarrow \quad -\frac{3}{4} = \frac{9}{4c} \quad \therefore \; c = -3$

14. (a) : Area of ΔPQR,



$A = \frac{1}{2}x\sqrt{d^2 - x^2}$

For max or min, $\frac{dA}{dx} = 0$

$$\Rightarrow \quad \frac{1}{2}\sqrt{d^2-x^2}+\frac{1}{2}x\left(\frac{-2x}{2\sqrt{d^2-x^2}}\right)=0$$

$$\Rightarrow \quad \frac{d^2-x^2-x^2}{2\sqrt{d^2-x^2}}=0 \;\Rightarrow\; x=\frac{d}{\sqrt{2}}$$

Now, $$\frac{d^2A}{dx^2}=\frac{(-4x)\cdot 2\sqrt{d^2-x^2}-2\dfrac{(-2x)(d^2-2x^2)}{2\sqrt{d^2-x^2}}}{4(d^2-x^2)}$$

At $x=d/\sqrt{2}$, $$\frac{d^2A}{dx^2}=\frac{\dfrac{-8d}{\sqrt{2}}\cdot\dfrac{d}{\sqrt{2}}+\sqrt{2}d(0)}{4(d^2-d^2/2)}=-ve$$

∴ A is max at $x=d/\sqrt{2}$ then $y=d/\sqrt{2}$

∴ Area of ΔPQR is maximum when it is isosceles.

15. (b) : $(x-1)^2+y^2=1, r=1 \Rightarrow b=2$
and $x^2+(y-2)^2=4, r=2 \Rightarrow a=4$

$$\Rightarrow \quad \frac{x^2}{16}+\frac{y^2}{4}=1 \Rightarrow x^2+4y^2=16$$

16. (d) : The equation of the line passing through $(3, b, 1)$ and $(5, 1, a)$ is

$$\frac{x-5}{2}=\frac{y-1}{1-b}=\frac{z-a}{a-1}=\mu \text{ (say)}$$

The line crosses the yz plane where $x = 0$, *i.e.*

$$-5=2\mu \Rightarrow \mu=-\frac{5}{2}$$

Again $y=\mu(1-b)+1=\dfrac{17}{2}$

$$\Rightarrow \quad -\frac{5}{2}(1-b)+1=\frac{17}{2} \Rightarrow -\frac{5}{2}(1-b)=\frac{15}{2}$$

$\Rightarrow \quad (1-b)=-3 \quad ∴ \quad b=4$

Again $z=\mu(a-1)+a=-\dfrac{13}{2}$

$$\Rightarrow \quad -\frac{5}{2}(a-1)+a=-\frac{13}{2} \Rightarrow -\frac{3}{2}a+\frac{5}{2}=-\frac{13}{2}$$

$$\Rightarrow \quad -\frac{3}{2}a=-9 \Rightarrow a=6$$

17. (a) : Series A = 101, 102, 103, ..., 200
Series B = 151, 152, 153, ..., 250
Series B is obtained by adding a fixed quantity to each item of series A, we know that variance is independent of change of origin. So both series have the same variance so ratio of their variances is 1.

18. (a) : Given, $\dfrac{\vec{v}\cdot\vec{u}}{|\vec{u}|}=\dfrac{\vec{u}\cdot\vec{w}}{|\vec{u}|}$ and $\vec{v}\cdot\vec{w}=0$

Also $|\vec{u}|=1$, $|\vec{v}|=2$, $|\vec{w}|=3$

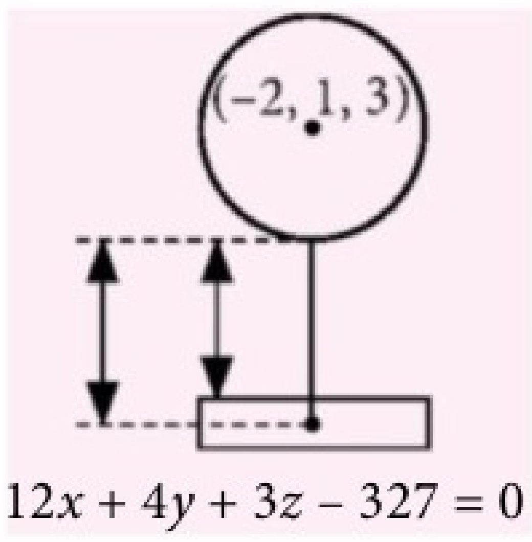
Now, $|\vec{u}-\vec{v}+\vec{w}|^2=\vec{u}^2+\vec{v}^2+\vec{w}^2-2\vec{u}\cdot\vec{v}-2\vec{v}\cdot\vec{w}+2\vec{u}\cdot\vec{w}$

$= 1 + 4 + 9 + 0 = 14$

19. (b) : Let's prepare the truth table

p	q	$\sim q$	$p \leftrightarrow q$	$p \leftrightarrow \sim q$	$\sim(p\leftrightarrow \sim q)$
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	F	T

As the column for $\sim(p \leftrightarrow \sim q)$ and $(p \leftrightarrow q)$ is the same, we conclude that $\sim(p \leftrightarrow \sim q)$ is equivalent to $(p \leftrightarrow q)$.
$\sim(p \leftrightarrow \sim q)$ is not a tautology.



20. (b) : In order to determine the shortest distance between the plane and sphere, we find the distance from the centre of sphere to the plane – Radius of sphere

∴ Centre of sphere is (–2, 1, 3)

Required distance is

$$\frac{|-24+4+9-327|}{\sqrt{12^2+4^2+3^2}}-\sqrt{2^2+1^2+3^2+155}$$

= 26 – 13 = 13 units.

21. (0) : $$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=0$$

$$\Rightarrow \quad \begin{vmatrix} z & z & z \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=0$$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$$\Rightarrow \quad z\begin{vmatrix} 1 & 1 & 1 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=0$$

$\Rightarrow \quad z\{(z+\omega^2)(z+\omega)-1-\omega(z+\omega-1)+\omega^2$
$(1-z-\omega^2)\}=0$ which gives $z^3=0$

Thus $z = 0$ is the only distinct solution.

22. (216) : We 1st note that a number is divisible by 3 if the sum of the digits is divisible by 3. Hence from the given digits the eligible digits are 0, 1, 2, 4, 5 or 1, 2, 3, 4, 5.
Number of arrangements in 1st $=\lfloor\underline{5}-\lfloor\underline{4}$ and in 2nd is $\lfloor\underline{5}$.
∴ Total number of desired numbers is 96 + 120 = 216.

23. (1.4) : Since, $\cos\alpha+\cos\beta=\dfrac{3}{2}$

$$\Rightarrow \quad 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{3}{2} \qquad \ldots(i)$$

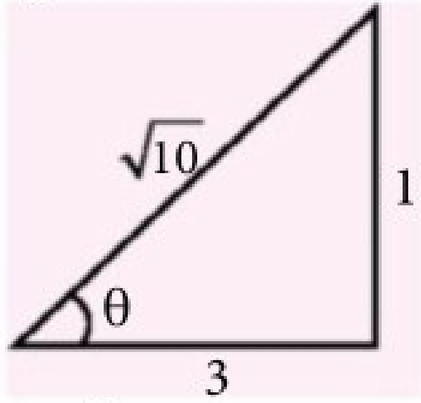
Also, $\sin\alpha+\sin\beta=\dfrac{1}{2}$

$$\Rightarrow \quad 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{1}{2} \qquad \text{...(ii)}$$

Dividing (ii) by (i), $\tan\left(\frac{\alpha+\beta}{2}\right)=\frac{1}{3}$

Since θ is the arithmetic mean of α and β.

$$\Rightarrow \quad \theta = \frac{\alpha+\beta}{2} \quad \Rightarrow \quad \tan\theta=\frac{1}{3}$$



$$\therefore \quad \sin\theta=\frac{1}{\sqrt{10}} \text{ and } \cos\theta=\frac{3}{\sqrt{10}}$$

Now, $\sin 2\theta + \cos 2\theta = 2\sin\theta\cos\theta + 2\cos^2\theta - 1$

$$=2\times\frac{1}{\sqrt{10}}\times\frac{3}{\sqrt{10}}+2\left(\frac{9}{10}\right)-1=\frac{6}{10}+\frac{18}{10}-1=\frac{7}{5}=1.4$$

24. (512) : Let first term of a G.P. is a and common ratio is r.

$\therefore \quad t_5 = ar^4 = 2$ [Given]

$$\therefore \quad \prod_{i=1}^{9} t_i = a\cdot ar\cdot ar^2 \ldots ar^8 = a^9 r^{\frac{8\times 9}{2}} = a^9 r^{36} = (ar^4)^9$$

$$= 2^9 = 512$$

25. (0.14) : We have, $P(\text{I})=0.3, P(\bar{\text{I}})=1-0.3=0.7,$

$P(\text{II})=0.2, P(\overline{\text{II}})=1-0.2=0.8$

Required probability $= P(\bar{\text{I}}\cap\text{II})=P(\bar{\text{I}})P(\text{II})$

$$=(0.7)(0.2)=0.14.$$

◆◆

EXAM ALERT

JOINT ENTRANCE EXAMINATION MAIN – 2020

The National Testing Agency (NTA) will conduct the JEE Main-2020 Examination twice for admission to Undergraduate Programs in NITs, IIITs and other Centrally Funded Technical Institutions (CFTIs), etc. in the next academic year (2020-2021).The First JEE Main-2020 will be conducted in January 2020 between 6th January (Monday) and 11th January (Saturday) 2020 and the Second JEE Main-2020 will be conducted between 3rd April (Friday) and 9th April (Thursday) 2020.

As per the eligibility criteria for B.Arch and B.Planning courses and according to the opinion of Subject Experts, a few changes in the pattern of the question paper(s) and number of question(s) for B.E./B.Tech, B.Arch and B.Planning have been approved by the JEE Apex Board (JAB) for the conduct of JEE Main-2020 Examination.

The pattern of question paper(s) is given below:

PAPER	SUBJECTS	NO. OF QUESTIONS	TYPE OF QUESTIONS	TIMING OF THE EXAMINATION(IST)	
				First Shift	Second Shift
B.E./B.Tech.	Mathematics	25(20+5)	20 Objective Type - Multiple Choice Questions (MCQs) & 5 Questions with answer as numerical value, with equal weightage to Mathematics, Physics & Chemistry	09:30 a.m. to 12:30 p.m.	02:30 p.m. to 05:30 p.m.
	Physics	25(20+5)			
	Chemistry	25(20+5)			
B. Arch.	Mathematics – Part I	25(20+5)	20 Objective Type - Multiple Choice Questions (MCQs) & 5 Questions for which answer is a numerical value	09:30 a.m. to 12:30 p.m.	02:30 p.m. to 05:30 p.m.
	Aptitude Test – Part II	50	Objective Type - Multiple Choice Questions (MCQs)		
	Drawing Test – Part III	2	Questions for drawing test		
B. Planning	Mathematics – Part I	25(20+5)	20 Objective Type - Multiple Choice Questions (MCQs) & 5 Questions for which answer is a numerical value	02:30 p.m. to 05:30 p.m.	
	Aptitude Test – Part II	50	Objective Type - Multiple Choice Questions (MCQs)		
	Planning Based Questions - Part III	25	Objective Type - Multiple Choice Questions (MCQs)		
Note: The timing for those Candidates who opt for B.Arch. and B.Planning (both) will be 02:30 p.m. to 06:00 p.m. (IST)					

The above Examinations will be held in "Computer Based Test" (CBT) Mode only, except that the Drawing Test for B.Arch. will be held in "Pen & Paper" (offline) mode. A candidate may appear in B.E./B.Tech, B.Arch and B.Planning depending upon the course/s he/she wishes to pursue.

The candidates aspiring to take admission to the undergraduate programs at IITs for the year 2020 will also have to appear in B. E. /B. Tech. Paper of JEE Main -2020. Based on the performance in the B.E./B. Tech. of JEE Main-2020, number of top candidates as per the requirement for JEE Advanced-2020 will be eligible to appear in JEE Advanced-2020. Admission to IITs will be based on category -wise All India Rank (AIR) in JEE Advanced,subject to the conditions as would be mentioned in JEE Advanced-2020 website.

For the April JEE Main-2020, a separate notice will be issued later on and the candidates will be required to apply separately. However, candidates are not required to compulsorily appear in both the tests i.e. January JEE Main-2020 and April JEE Main-2020. In case, a candidate appears in both the tests, the better of the two scores will be used for the admissions and eligibility of JEE Advanced-2020.

For more information visit www.jeemain.nic.in

STRAIGHT LINES

STRAIGHT LINES

$ax + by + c = 0$, is the general equation of a straight line, where a, b, c are real numbers and a or b can be zero, but not both at the same time. Slope of the line $= m = \tan\theta = -a/b$.

Different Forms of Equation of Straight Line

Slope-intercept form : $y = mx + c$, where m is the slope and c is an intercept on y-axis.

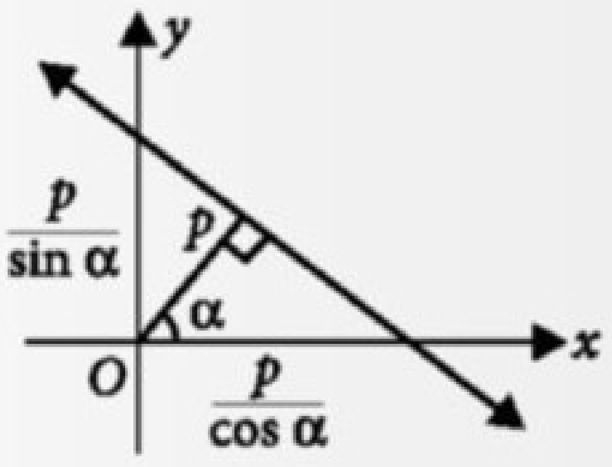
Point-slope form : $\frac{(y-y_1)}{(x-x_1)} = m$, where m is the slope and (x_1, y_1) be given point on the line.

Two point form : $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$, where (x_1, y_1) and (x_2, y_2) are two given points on the line.

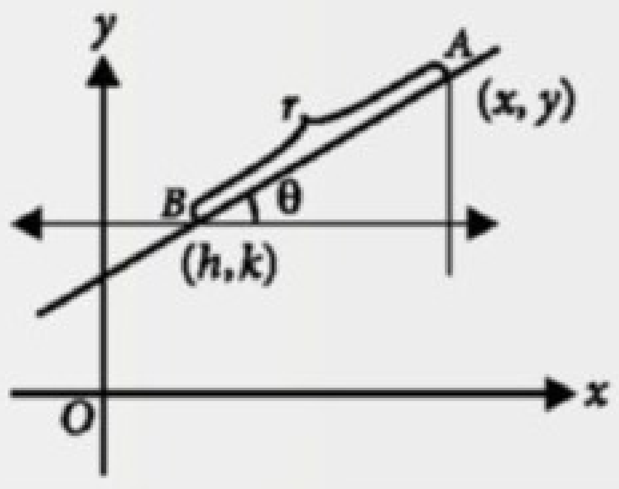
Intercept form : $\frac{x}{a}+\frac{y}{b}=1$, where a and b represents intercepts on x and y axes respectively by the line.

Determinant form : $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$, where (x_1, y_1) and (x_2, y_2) are any two points on the line.

Perpendicular form (or) Normal form :
$x\cos\alpha + y\sin\alpha = p$



Parametric form : $\frac{x-h}{\cos\theta}=\frac{y-k}{\sin\theta}=r$ where r is the distance between A and B and $x = h + r\cos\theta, y = k + r\sin\theta$



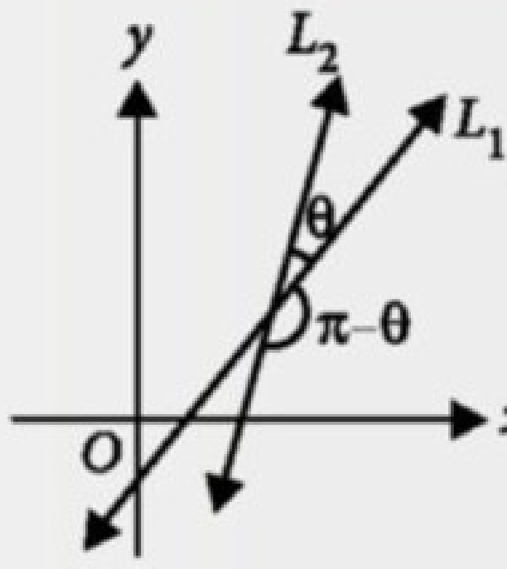
Angle Bisectors

et $a_1 x + b_1 y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ be two lines then quation of angle bisectors is given by

$$a_1x+b_1y+c_1 \over \sqrt{a_1^2+b_1^2}\Bigg| = \left|\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right|$$

Angle between Two Lines

For $L_1 : a_1 x + b_1 y + c_1 = 0$ and $L_2 : a_2 x + b_2 y + c_2 = 0$, acute angle (θ) between two lines L_1 and L_2 is given by $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$, where m_1 = slope of L_1, m_2 = slope of L_2.



- L_1 and L_2 are perpendicular iff $m_1m_2 = -1$.
- L_1 and L_2 are parallel iff $m_1 = m_2$.

Perpendicular Distance

- Distance of a point (x_1, y_1) from line $ax + by + c = 0$ is give by $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$.
- Distance between two parallel lines $ax + by + c_1 = 0$ an $ax + by + c_2 = 0$ is given by $\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$.

Image of a Point and Foot of Perpendicular from a Poin

- The image of (x_1, y_1) in the line $ax + by + c = 0$ is give by (x_2, y_2), if $\frac{x_2-x_1}{a} = \frac{y_2-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2}$.
- If (x_2, y_2) be the foot of the perpendicular from (x_1, y_1) t the line $ax + by + c = 0$, then $\frac{x_2-x_1}{a} = \frac{y_2-y_1}{b} = \frac{-(ax_1+by_1+c)}{a^2+b^2}$.

Family of Lines

Let $L_1 : a_1x + b_1y + c_1 = 0$ and $L_2 : a_2x + b_2 y + c_2 = 0$ be tw lines, then $L_1 + \lambda L_2 = 0$ or $L_2 + \lambda L_1 = 0$ gives family of line passing through the point of intersection of L_1 and L_2.

Condition of Concurrency of Three Lines

- If $L_r : a_r x + b_r y + c_r = 0$ (where $r = 1, 2, 3$) then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$
- $\lambda_1L_1 + \lambda_2L_2 + \lambda_3L_3 = 0$, where $\lambda_1, \lambda_2, \lambda_3 \in R$, not all zeros.

DIFFERENTIAL EQUATIONS

CONCEPT
MAP

Class XII

DIFFERENTIAL EQUATIONS

An equation involving dependent variable, independent variable and derivative(s) of dependent variable(s) w.r.t. independent variable(s) is called differential equation.

Solution of Differential Equation

function of the form $y = f(x) + c$, which satisfies the iven differential equation is the solution of the differential quation.

- **General Solution :** Contains as many arbitrary constants as order of equation.
- **Particular Solution :** Obtained by assigning values to arbitrary constants.

Methods of Solving First Order and First Degree Differential Equations

Variable Separable

- If $\dfrac{dy}{dx} = f(x)g(y)$, then $\displaystyle\int \frac{dy}{g(y)} = \int f(x)\,dx + c.$
- If $\dfrac{dy}{dx} = f(ax + by + c)$, then put $ax + by + c = z$ and $b\dfrac{dy}{dx} = \dfrac{dz}{dx} - a$, then apply variable separable method.

Homogeneous Differential Equation

f $\dfrac{dy}{dx} = \dfrac{f(x, y)}{g(x, y)}$, where $f(x, y)$, $g(x, y)$ are homogeneous unctions of the same degree in x and y, then put $y = vx$ and $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$, then apply variable separable method.

Linear Differential Equation

f $\dfrac{dy}{dx} + Py = Q$, where P, Q are functions of x, then $ye^{\int Pdx} = \int Qe^{\int Pdx}dx + c$, where $e^{\int Pdx}$ is the integrating actor (I.F.).

Order and Degree of Differential Equation

Order : The order of a differential equation is the order of it highest order derivative.

Degree : Degree is the highest power of the highest orde derivative in a polynomial equation of differentials.

Formation of Differential Equation

- Differentiate the given equation as many times as th number of arbitrary constants and then eliminate th arbitrary constants from them.
- Order of differential equation = Number of arbitrar constants in differential equation.

Orthogonal Trajectory

Any curve which meet each member of a given family of curve at right angle is called an orthogonal trajectory of the family.

Steps to Find the Orthogonal Trajectory :

- Let $g(x, y, c) = 0$ be the equation of the given family of curve
- Differentiate the equation $g(x, y, c) = 0$ w.r.t. x and eliminat the parameter c.
- Put $-\dfrac{dx}{dy}$ for $\dfrac{dy}{dx}$ in the differential equation so obtained
- Now, solve this differential equation to get the orthogona trajectories.

Method of Solving Differential Equations of Second Order

Consider a differential equation of second order.

$$A_0\frac{d^2y}{dx^2} + A_1\frac{dy}{dx} + A_2y = 0 \qquad \text{...(}$$

Putting $\dfrac{d}{dx} = D$ in (i), we get

$$(A_0D^2 + A_1D + A_2)y = 0 \qquad \text{...(i}$$

Auxiliary Equation (A.E.) is given by

$$A_0D^2 + A_1D + A_2 = 0 \qquad \text{...(ii}$$

which is a quadratic equation in D.

Let roots of (iii) be given by $D = C_1, C_2$

Roots	Solution
$C_1 \neq C_2$	$y = Ae^{C_1x} + Be^{C_2x}$
$C_1 = C_2(C)$	$y = (A + Bx)e^{Cx}$

CONCEPT BOOSTER

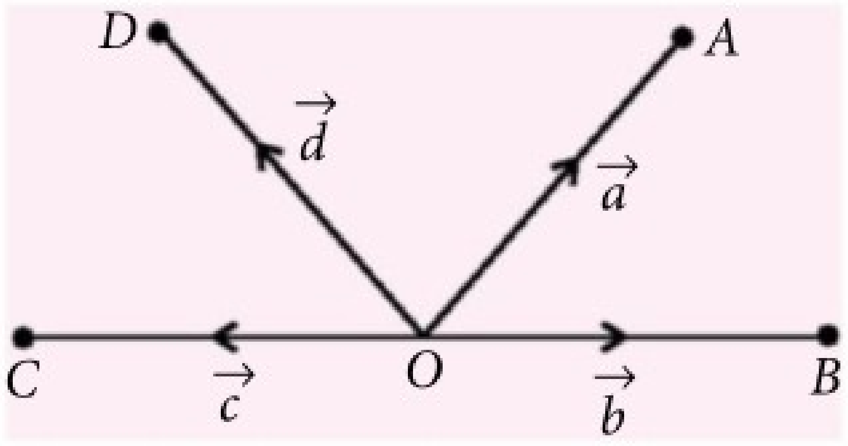
Vectors and Three-Dimensional Geometry

Vectors

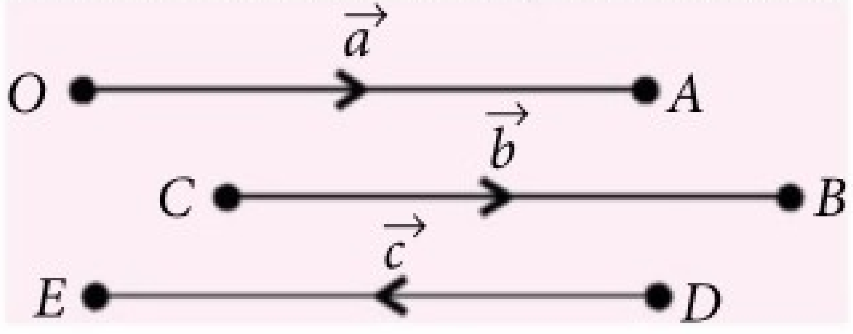
A vector may be described as a quantity having both magnitude and direction. A vector is generally represented by a directed line segment, say $\overrightarrow{AB}$. A is called the initial point and B is called the terminal point. The magnitude of vector $\overrightarrow{AB}$ is expressed by $|\overrightarrow{AB}|$.

TYPES OF VECTORS

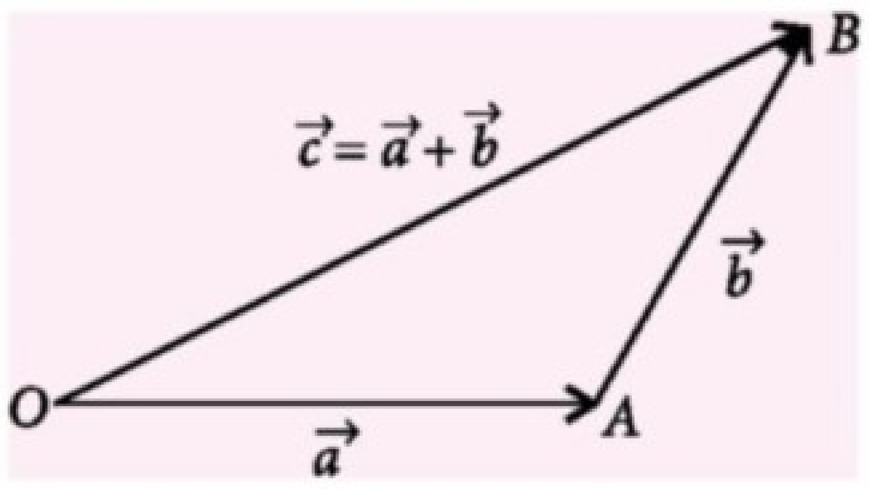
- **Position Vector:** If P is any point in the space, then the vector $\overrightarrow{OP}$ is called position vector of point P, where O is the origin of reference. Thus for any two points A and B in the space, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$.
- **Co-initial Vectors:** Vectors having same initial point are called co-initial vectors.



  Here $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ and $\overrightarrow{OD}$ are co-initial vectors.
- **Equal Vectors :** Two vectors are said to be equal if and only if they have equal magnitudes and same direction.
- **Zero Vector (Null Vector):** A vector whose initial and terminal points are same, is called the null vector. Such vector has zero magnitude and no direction, and denoted by $\vec{0}$.
- **Like and Unlike Vectors:** Two vectors are said to be
  - (i) Like, when they are in same direction.
  - (ii) Unlike, when they are in opposite directions. $\vec{a}$ and $-\vec{a}$ are two unlike vectors as their directions are opposite, $\vec{a}$ and $3\vec{a}$ are like vectors.
- **Unit Vector :** A unit vector is a vector whose magnitude is unity. We write, unit vector in the direction of $\vec{a}$ as $\hat{a}$. Therefore $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$.
- **Parallel Vectors:** Two or more vectors are said to be parallel, if they have the same support or parallel support. Parallel vectors may have equal or unequal magnitudes and direction may be same or opposite.



ADDITION OF TWO VECTORS



Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{AB} = \vec{b}$ and $\overrightarrow{OB} = \vec{c}$.

Here $\vec{c}$ is sum (or resultant) of vectors $\vec{a}$ and $\vec{b}$. It is to be noticed that the initial point of $\vec{b}$ coincides with the terminal point of $\vec{a}$ and the line joining the initial point of $\vec{a}$ to the terminal point of $\vec{b}$ represents vector $\vec{a} + \vec{b}$ in magnitude and direction.

PROPERTIES

- $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (Vector addition is commutative)
- $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$ (Vector addition is associative)
- $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$, equality holds when $\vec{a}$ and $\vec{b}$ are like vectors.
- $|\vec{a} + \vec{b}| \ge ||\vec{a}| - |\vec{b}||$, equality holds when $\vec{a}$ and $\vec{b}$ are unlike vectors.
- $\vec{a} + \vec{0} = \vec{a} = \vec{0} + \vec{a}$
- $\vec{a} + (-\vec{a}) = \vec{0} = (-\vec{a}) + \vec{a}$

MULTIPLICATION OF VECTOR BY SCALARS

If $\vec{a}$ is a vector and m is a scalar, then $m\vec{a}$ is a vector parallel to $\vec{a}$ whose magnitude is $|m|$ times that of $\vec{a}$. This multiplication is called scalar multiplication. If $\vec{a}$ and $\vec{b}$ are vectors and m, n are scalars, then :

- $m(\vec{a}) = (\vec{a})m = m\vec{a}$
- $m(n\vec{a}) = n(m\vec{a}) = (mn)\vec{a}$
- $(m+n)\vec{a} = m\vec{a} + n\vec{a}$
- $m(\vec{a}+\vec{b}) = m\vec{a} + m\vec{b}$

SECTION FORMULA

If $\vec{a}$ and $\vec{b}$ are the position vectors of two points A and B, then the position vector of a point which divides AB in the ratio $m : n$ is given by $\vec{r} = \dfrac{n\vec{a}+m\vec{b}}{m+n}$.

Note : Position vector of mid-point of $AB = \dfrac{\vec{a}+\vec{b}}{2}$.

SCALAR PRODUCT OF TWO VECTORS (DOT PRODUCT)

The scalar product, $\vec{a}\cdot\vec{b}$ of two non-zero vectors $\vec{a}$ and $\vec{b}$ is defined as $|\vec{a}||\vec{b}|\cos\theta$, $0 \le \theta \le \pi$ where θ is the angle between the two vectors, when drawn with same initial point.

If at least one of $\vec{a}$ and $\vec{b}$ is a zero vector, then $\vec{a}\cdot\vec{b}$ is defined as zero.

PROPERTIES

- $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ (Scalar product is commutative)
- $\vec{a}^2 = \vec{a}\cdot\vec{a} = |\vec{a}|^2$
- $(m\vec{a})\cdot\vec{b} = m(\vec{a}\cdot\vec{b}) = \vec{a}\cdot(m\vec{b})$, where m is a scalar.
- $\theta = \cos^{-1}\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)$
- $\vec{a}\cdot\vec{b} = 0 \Leftrightarrow$ Vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other. [Provided $\vec{a}, \vec{b}$ are non-zero vectors]
- $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$
- $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
- $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$
- $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$
- Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$.
  Then, $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$
- Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

VECTOR PRODUCT OF TWO VECTORS (CROSS PRODUCT)

If $\vec{a}$ and $\vec{b}$ are two vectors and θ is the angle between them, then $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$, where $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{a}$, $\vec{b}$ and $\hat{n}$ forms a right handed screw system .

PROPERTIES

- $\vec{a}\times\vec{b} = 0 \Leftrightarrow \vec{a}$ and $\vec{b}$ are parallel (collinear) $(\vec{a} \ne 0,\ \vec{b} \ne 0)$ *i.e.*, $\vec{a} = K\vec{b}$, where K is a scalar.
- $\vec{a}\times\vec{b} \ne \vec{b}\times\vec{a}$ (Not commutative)
- $(m\vec{a})\times\vec{b} = \vec{a}\times(m\vec{b}) = m(\vec{a}\times\vec{b})$, where m is a scalar.
- $\vec{a}\times(\vec{b}+\vec{c}) = (\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c})$ (Distributive)
- $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$
- $\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{i} = \hat{j}$
- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$.
- Geometrically $|\vec{a}\times\vec{b}|$ = area of the parallelogram whose two adjacent sides are represented by $\vec{a}$ and $\vec{b}$.
- Area of any quadrilateral whose diagonal vectors are $\vec{d}_1$ and $\vec{d}_2$ is given by $\dfrac{1}{2}|\vec{d}_1\times\vec{d}_2|$.
- Unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$ is $\hat{n} = \pm\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$.
- A vector of magnitude r and perpendicular to the plane of $\vec{a}$ and $\vec{b}$ is $\pm\dfrac{r(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$.
- If θ is the angle between $\vec{a}$ and $\vec{b}$, then $\sin\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$.
- **Lagrange's identity :** For any two vectors $\vec{a}$ and $\vec{b}$; $(\vec{a}\times\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} \end{vmatrix}$
- **Formulation of vector product in terms of scalar product**
  The vector product $\vec{a}\times\vec{b}$ is the vector $\vec{c}$, such that
  (i) $|\vec{c}| = \sqrt{|\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2}$
  (ii) $\vec{c}\cdot\vec{a} = 0,\ \vec{c}\cdot\vec{b} = 0$
  (iii) $\vec{a}, \vec{b}, \vec{c}$ form a right handed system

AREA OF TRIANGLE

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the position vectors of three points A, B and C, then the area of triangle $ABC = \dfrac{1}{2}\left[\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}\right]$.

The points A, B and C are collinear if

$\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} = 0$

SCALAR TRIPLE PRODUCT

The scalar triple product of three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is defined as $(\vec{a}\times\vec{b})\cdot\vec{c} = |\vec{a}||\vec{b}||\vec{c}|\sin\theta\cos\phi$

where, θ is the angle between $\vec{a}$ and $\vec{b}$ and ϕ is the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$. It is also defined as $[\vec{a}\ \vec{b}\ \vec{c}]$, spelled as box product.

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, then $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$.

PROPERTIES

- In a scalar triple product, the position of dot and cross can be interchanged.
  i.e., $\vec{a}\cdot(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\cdot\vec{c}$
  or $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$
- $\vec{a}\cdot(\vec{b}\times\vec{c}) = -\vec{a}\cdot(\vec{c}\times\vec{b})$
- Scalar triple product geometrically represents the volume of the parallelepiped whose three coterminous edges are represented by $\vec{a}$, $\vec{b}$ and $\vec{c}$ *i.e.,* $V = [\vec{a}\ \vec{b}\ \vec{c}]$.
- Vectors $\vec{a}, b, \vec{c}$ are coplanar $\Leftrightarrow$ $[\vec{a}\ \vec{b}\ \vec{c}] = 0$.
- Scalar product of three vectors, two of which are equal or parallel is 0.
- If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors, then $[\vec{a}\ \vec{b}\ \vec{c}] > 0$ for right handed system and $[\vec{a}\ \vec{b}\ \vec{c}] < 0$ for left handed system.
- $[\hat{i}\ \hat{j}\ \hat{k}] = 1$
- $[K\vec{a}\ \vec{b}\ \vec{c}] = K[\vec{a}\ \vec{b}\ \vec{c}]$
- $[(\vec{a}+\vec{b})\ \vec{c}\ \vec{d}] = [\vec{a}\ \vec{c}\ \vec{d}] + [\vec{b}\ \vec{c}\ \vec{d}]$
- The volume of the tetrahedron *OABC* with *O* as origin and the position vectors of *A*, *B* and *C* being $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively is given by $V = \frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$.
- The position vector of the centroid of a tetrahedron if the position vectors of its angular vertices are $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are given by $\frac{1}{4}[\vec{a} + \vec{b} + \vec{c} + \vec{d}]$.

VECTOR TRIPLE PRODUCT

The vector triple product of three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is defined as $\vec{a}\times(\vec{b}\times\vec{c})$. If at least one $\vec{a}, \vec{b}$ and $\vec{c}$ is a zero vector or $\vec{b}$ and $\vec{c}$ are collinear vectors or $\vec{a}$ is perpendicular to both $\vec{b}$ and $\vec{c}$, only then $\vec{a}\times(\vec{b}\times\vec{c}) = \vec{0}$.

In all other cases $\vec{a}\times(\vec{b}\times\vec{c})$ will be a non-zero vector in the plane of non-collinear vectors and perpendicular to the vector $\vec{a}$.

Thus we can take $\vec{a}\times(\vec{b}\times\vec{c}) = \lambda\vec{b} + \mu\vec{c}$, for some scalars λ and μ.

Since, $\vec{a} \perp \vec{a}\times(\vec{b}\times\vec{c}) \Rightarrow a\cdot(\vec{a}\times(\vec{b}\times\vec{c})) = 0$

$\Rightarrow \lambda(\vec{a}\cdot\vec{b}) + \mu(\vec{a}\cdot\vec{c}) = 0$

$\Rightarrow \lambda = (\vec{a}\cdot\vec{c})\alpha,\ \mu = -(\vec{a}\cdot\vec{b})\alpha$, for some scalar α.

Hence, $\vec{a}\times(\vec{b}\times\vec{c}) = \alpha[(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c})]$, for any vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfying the conditions given in the beginning. In particular if we take, $\vec{a} = \vec{b} = \hat{i},\ \vec{c} = \hat{j}$, then $\alpha = 1$.

Hence, $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

SCALAR AND VECTOR PRODUCT OF FOUR VECTORS

- If $\vec{p}, \vec{q}, \vec{r}, \vec{s}$ are four vectors then the scalar product is computed as $(\vec{p}\times\vec{q})\cdot(\vec{r}\times\vec{s})$ and defined by
  $$(\vec{p}\times\vec{q})\cdot(\vec{r}\times\vec{s}) = \begin{vmatrix} \vec{p}\cdot\vec{r} & \vec{p}\cdot\vec{s} \\ \vec{q}\cdot\vec{r} & \vec{q}\cdot\vec{s} \end{vmatrix}$$
- Vector product of four vectors are computed as $(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{s}) = [\vec{p}\ \vec{q}\ \vec{s}]\vec{r} - [\vec{p}\ \vec{q}\ \vec{r}]\vec{s}$
  $= [\vec{p}\ \vec{r}\ \vec{s}]\vec{q} - [\vec{q}\ \vec{r}\ \vec{s}]\vec{p}$

LINEAR COMBINATIONS

Given a finite set of vectors $\vec{a}, \vec{b}, \vec{c}, \ldots$, then the vector $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c} + \ldots$ is called a linear combination of $\vec{a}, \vec{b}, \vec{c}, \ldots$ for any $x, y, z, \ldots \in R$.

- **Fundamental theorem in plane :** Let $\vec{a}, \vec{b}$ be non-zero, non-collinear vectors. Then any vector $\vec{r}$ coplanar with $\vec{a}, \vec{b}$ can be expressed uniquely as a linear combination of $\vec{a}, \vec{b}$ *i.e.,* there exist some unique $x, y \in R$ such that $x\vec{a} + y\vec{b} = \vec{r}$.
- **Fundamental theorem in space :** Let $\vec{a}, \vec{b}, \vec{c}$ be non-zero, non-coplanar vectors in space. Then any vector $\vec{r}$ can be uniquely expressed as a linear combination of $\vec{a}, \vec{b}, \vec{c}$ *i.e.,* there exist some unique $x, y, z \in R$ such that $x\vec{a} + y\vec{b} + z\vec{c} = \vec{r}$.
- **Linearly independent and dependent vectors**
  (i) If $\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n$ are n non-zero vectors, and $k_1, k_2, \ldots, k_n$ are n scalars and if the linear combination $k_1\vec{x}_1 + k_2\vec{x}_2 + \ldots + k_n\vec{x}_n = 0$
  $\Rightarrow k_1 = 0, k_2 = 0, \ldots, k_n = 0$
  then we say that vectors $\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n$ are linearly independent vectors.

(ii) If $\vec{x}_1, \vec{x}_2, ..., \vec{x}_n$ are not linearly independent, then they are said to be linearly dependent vectors. *i.e.*, if $k_1\vec{x}_1 + k_2\vec{x}_2 + ...+ k_n\vec{x}_n = 0$ and if there exists at least one $k_r \neq 0$, then $\vec{x}_1, \vec{x}_2, ..., \vec{x}_n$ are said to be linearly dependent vectors.

- If $\vec{a} = 3\hat{i} + 2\hat{j} + 5\hat{k}$, then $\vec{a}$ is expressed as a linear combination of vectors $\hat{i}, \hat{j}, \hat{k}$. Also, $\vec{a}, \hat{i}, \hat{j}, \hat{k}$ form a linearly dependent set of vectors. In general, every set of four vectors forms a linearly dependent system.
- $\hat{i}, \hat{j}, \hat{k}$ are linearly independent set of vectors. For $K_1\hat{i} + K_2\hat{j} + K_3\hat{k} = 0 \Rightarrow K_1 = 0 = K_2 = K_3$.
- Two vectors $\vec{a}$ and $\vec{b}$ are linearly dependent *i.e.*, $\vec{a}$ is parallel to $\vec{b}$ *i.e.*, $\vec{a} \times \vec{b} = 0$. Conversely if $\vec{a} \times \vec{b} \neq 0$ then $\vec{a}$ and $\vec{b}$ are linearly independent.

COLLINEARITY AND COPLANARITY OF POINTS

The necessary and sufficient condition for three points with position vectors $\vec{a}, \vec{b}$ and $\vec{c}$ to be collinear is that there exist scalars *x*, *y*, *z* (not all zero) such that $x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$, where $x + y + z = 0$

The necessary and sufficient condition for four points with position vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ to be coplanar is that there exist scalars *x*, *y*, *z* ,*u* (not all zero) such that $x\vec{a} + y\vec{b} + z\vec{c} + u\vec{d} = \vec{0}$ where $x + y + z + u = 0$.

RECIPROCAL SYSTEM OF VECTORS

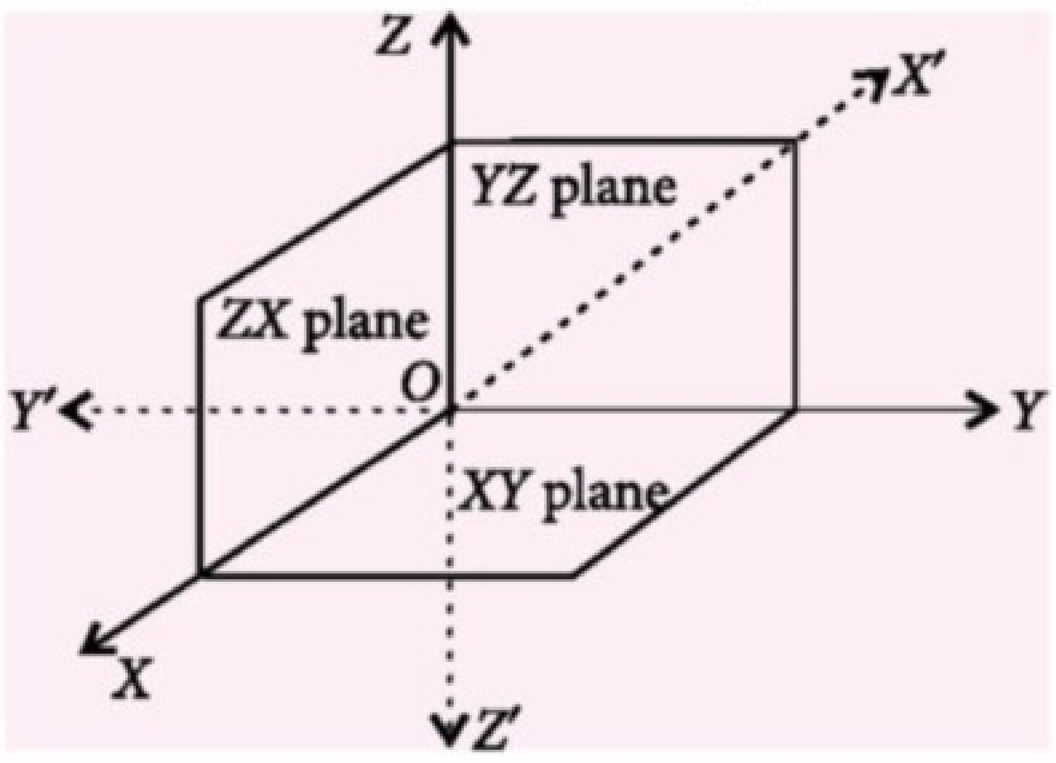
- $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are two sets of non-coplanar vectors such that $\vec{a} \cdot \vec{a}' = \vec{b} \cdot \vec{b}' = \vec{c} \cdot \vec{c}' = 1$, then the two systems are called reciprocal system of vectors.

Note : $\vec{a}' = \dfrac{\vec{b} \times \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$; $\vec{b}' = \dfrac{\vec{c} \times \vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$; $\vec{c}' = \dfrac{\vec{a} \times \vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$

Three - Dimensional Geometry

RECTANGULAR COORDINATE SYSTEM IN SPACE

Let '*O*' be any point in space and three lines are perpendicular to each other. These lines are known as coordinate axes and *O* is called origin. The planes *XY*, *YZ*, *ZX* are known as the coordinate planes.



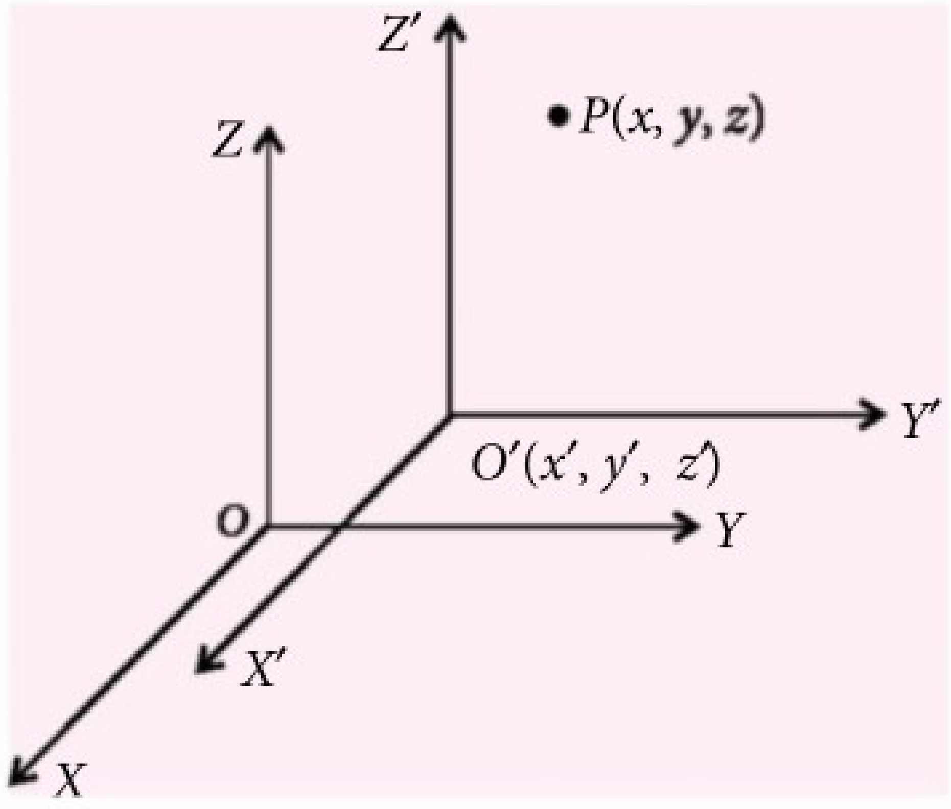
COORDINATES OF A POINT IN SPACE

Consider a point *P* in space whose position is given by triad (*x*, *y*, *z*) where *x*, *y*, *z* are perpendicular distance from *YZ*-plane, *ZX*-plane and *XY*-plane respectively.

If we assume $\hat{i}, \hat{j}, \hat{k}$ unit vectors along *OX*, *OY*, *OZ* respectively, then position vector of point *P* is $x\hat{i} + y\hat{j} + z\hat{k}$ or simply (*x*, *y*, *z*).

SHIFTING OF ORIGIN

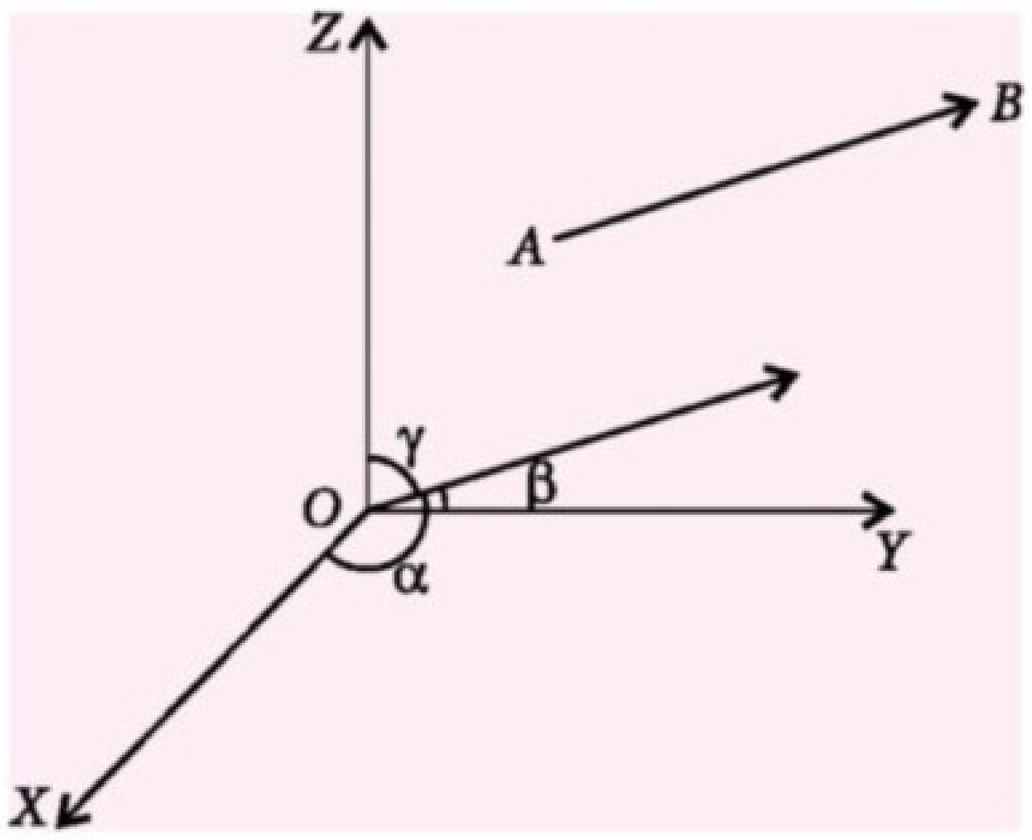
Shifting the origin to another point without changing the directions of the axes is called the translation of axes. Let the origin *O* be shifted to another point without changing the direction of axes.



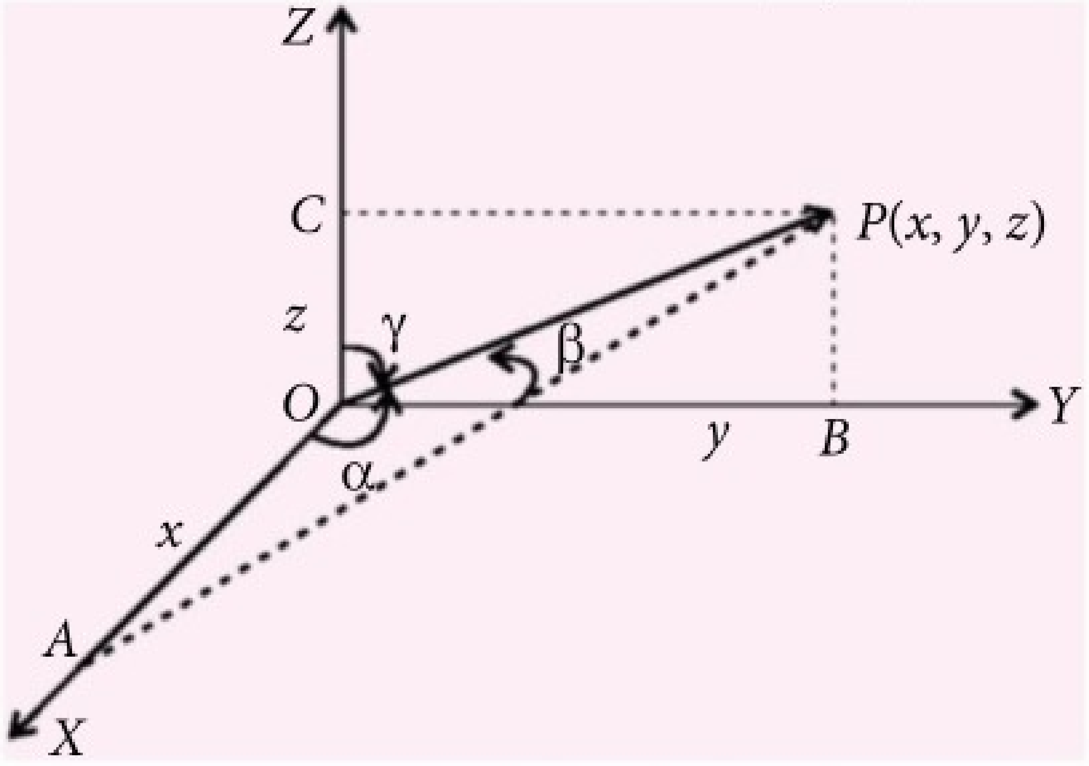
Let the new coordinate frame be *O*′*X*′*Y*′*Z*′. Let *P*(*x*, *y*, *z*) be a point with respect to the coordinate frame *OXYZ*. Then, coordinate of point *P* w.r.t. new coordinate frame *O*′ *X*′*Y*′*Z*′ is (x_1, y_1, z_1), where, $x_1 = x - x'$, $y_1 = y - y'$, $z_1 = z - z'$ where *O* is shifted at *O*′(*x*′, *y*′, *z*′).

DIRECTION COSINES & DIRECTION RATIOS OF A LINE

If α, β, γ be the angles which a given directed line makes with the positive direction of the *x*, *y*, *z* coordinate axes respectively, then cosα, cosβ, cosγ are called the direction cosines of the given line and are generally denoted by *l*, *m*, *n* respectively. Thus *l* = cosα, *m* = cosβ, *n* = cosγ. By definition it follows that the direction cosine of the *x*-axis is cos 0°, cos 90°, cos 90° *i.e.*, (1, 0, 0). Similarly direction cosines of the *y*-axis and *z*-axis are respectively (0, 1, 0) and (0, 0, 1).



- The unit vector along the line can be written as $l\hat{i}+m\hat{j}+n\hat{k}$.
- If d.c.'s of line AB is (l, m, n), then d.c.'s of line BA will be $(-l, -m, -n)$.
- Let OP be any line through the origin O which has direction cosines l, m, n. Let $P(x, y, z)$ and $OP = r$.



Then $OP^2 = x^2 + y^2 + z^2 = r^2$ ...(1)

From P draw PA, PB, PC perpendicular on the coordinate axes, so that $OA = x$, $OB = y$, $OC = z$

Also, $\angle POA = \alpha$, $\angle POB = \beta$ and $\angle POC = \gamma$.

From ΔAOP, $l=\cos\alpha=\frac{x}{r}\Rightarrow x=lr$

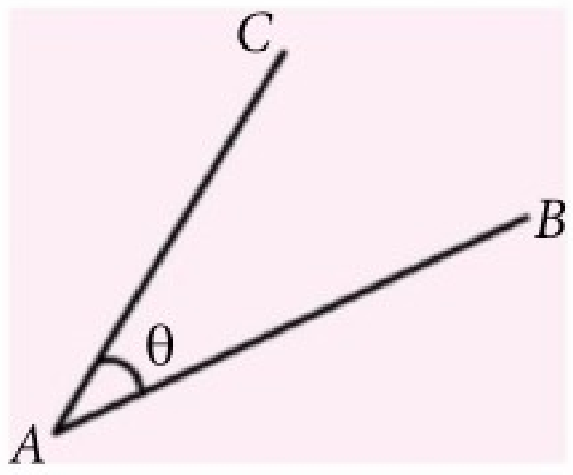
Similarly, $y = mr$ and $z = nr$

Hence from (1),

$r^2(l^2 + m^2 + n^2) = r^2 \quad \Rightarrow \quad l^2 + m^2 + n^2 = 1$

- If the coordinates of any point P be (x, y, z) and l, m, n be the direction cosines of the line OP, O being the origin, then (lr, mr, nr) will give us the coordinates of a point on the line OP which is at a distance r from $(0, 0, 0)$.

ANGLE BETWEEN TWO LINES



Let θ be the angle between two straight lines AB and AC whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2 respectively, is given by

$\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$.

If direction ratios of two lines are a_1, b_1, c_1 and a_2, b_2, c_2 are given, then angle between two lines is given

by $\cos\theta=\dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}$

Also, we have,

$$\sin^2\theta=1-\cos^2\theta=(l_1^2+m_1^2+n_1^2)(l_2^2+m_2^2+n_2^2)-(l_1l_2+m_1m_2+n_1n_2)^2$$

$$=(l_1m_2-l_2m_1)^2+(m_1n_2-m_2n_1)^2+(n_1l_2-n_2l_1)^2$$

$$\Rightarrow \sin\theta=\pm\sqrt{\sum(l_1m_2-l_2m_1)^2}$$

Note : (i) If the given lines are perpendicular, then $\theta = 90°$ *i.e.*, $\cos\theta = 0$

$\Rightarrow \quad l_1l_2 + m_1m_2 + n_1n_2 = 0$ or $a_1a_2 + b_1b_2 + c_1c_2 = 0$

(ii) If the given lines are parallel, then $\theta = 0°$ *i.e.*, $\sin\theta = 0$

$$\Rightarrow \frac{l_1}{l_2}=\frac{m_1}{m_2}=\frac{n_1}{n_2} \text{ or } \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

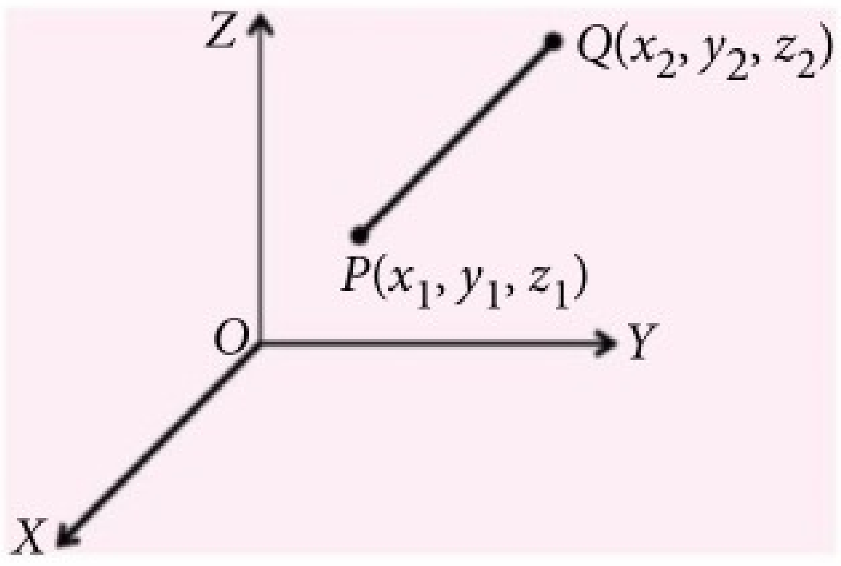
DIRECTION COSINES OF A LINE PASSING THROUGH TWO POINTS

The direction ratios of line PQ joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $x_2 - x_1 = a$(say), $y_2 - y_1 = b$(say) and $z_2 - z_1 = c$(say). Then direction cosines are

$$l=\pm\frac{(x_2-x_1)}{\sqrt{\sum(x_2-x_1)^2}},$$

$$m=\pm\frac{(y_2-y_1)}{\sqrt{\sum(y_2-y_1)^2}},$$

$$n=\pm\frac{(z_2-z_1)}{\sqrt{\sum(z_2-z_1)^2}}$$



THE STRAIGHT LINE

Straight line in three dimensional geometry is defined as intersection of two planes. So general equation of straight line is stated as the equations of both planes together *i.e.*, general equation of straight line is

$a_1x + b_1y + c_1z + d_1 = 0,\ a_2x + b_2y + c_2z + d_2 = 0$ ....(1)

So, equation (1) represents straight line which is obtained by intersection of two planes.

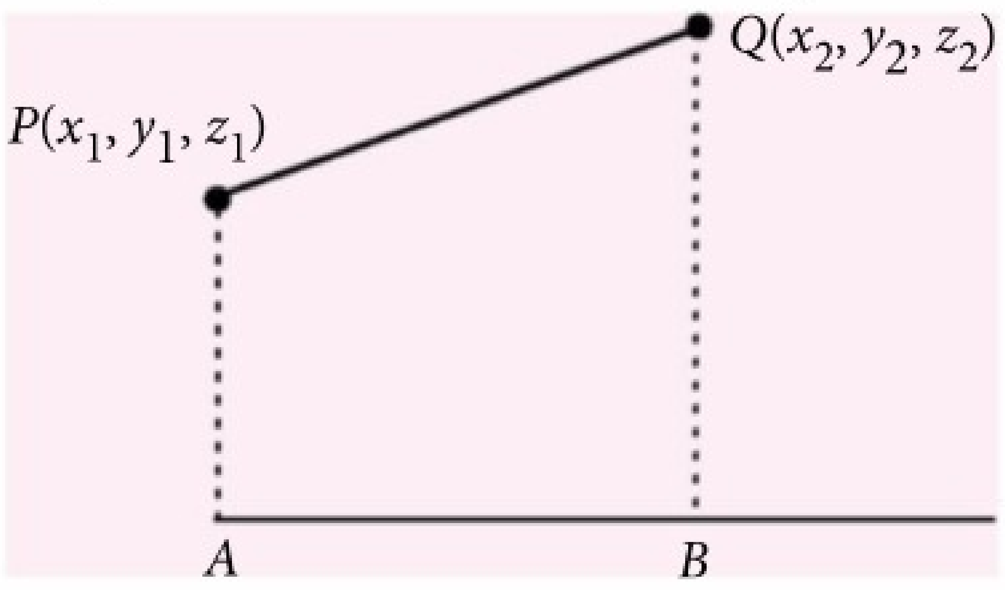
EQUATION OF STRAIGHT LINE IN DIFFERENT FORMS

- Equation of straight line passing through point $P(x_1, y_1, z_1)$ and whose direction cosines are l, m, n is $\dfrac{x-x_1}{l}=\dfrac{y-y_1}{m}=\dfrac{z-z_1}{n}=r$.
- Vector equation of straight line is $\vec{r}=\vec{r}_1+\lambda\vec{b}$ where $\vec{r}_1$ is the position vector of a point in the straight line and $\vec{b}$ is a vector parallel to the straight line.
- Equation of straight line passing through two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is
$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}.$$
- Equation of straight line passing through two points $P(\vec{r}_1)$ and $Q(\vec{r}_2)$ is $\vec{r}=\vec{r}_1+\lambda(\vec{r}_2-\vec{r}_1)$.

  Note : The general coordinates of a point on a line is given by $(x_1 + lr, y_1 + mr, z_1 + nr)$ where r is distance between the point (x_1, y_1, z_1) and point whose coordinates is to be written.

PROJECTION OF A LINE SEGMENT ON A GIVEN LINE

Projection of the line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ on another line whose direction cosines are l, m, n is $AB = |l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)|$. Here PQ and AB need not be coplanar.



COPLANARITY OF TWO LINES

Cartesian form : If the lines are

$$\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1} \text{ and } \frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$$

then condition of coplanarity is

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

and the equation of plane containing them is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0 \text{ or } \begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0.$$

Vector form : If the lines $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\lambda\vec{b}_2$ are coplanar, then $\left[\vec{a}_1\,\vec{b}_1\,\vec{b}_2\right]=\left[\vec{a}_2\,\vec{b}_1\,\vec{b}_2\right]$ and the equation of plane containing them is

$$\left[\vec{r}\,\vec{b}_1\,\vec{b}_2\right]=\left[\vec{a}_1\,\vec{b}_1\,\vec{b}_2\right] \text{ or } \left[\vec{r}\,\vec{b}_1\,\vec{b}_2\right]=\left[\vec{a}_2\,\vec{b}_1\,\vec{b}_2\right].$$

SHORTEST DISTANCE BETWEEN TWO SKEW LINES (NON-PARALLEL, NON-INTERSECTING LINES)

Two non-parallel, non-intersecting lines are called skew lines. The shortest distance between these two lines is the distance of the intercepting portion of a line perpendicular to both the lines.

Let the equation of two non-intersecting lines be

$$\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}=r_1(\text{say}) \quad ...(1)$$

$$\text{and } \frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}=r_2(\text{say}) \quad ...(2)$$

Any point on line (1) is $P(x_1 + l_1r_1, y_1 + m_1r_1, z_1 + n_1r_1)$ and on line (2) is $Q(x_2 + l_2r_2, y_2 + m_2r_2, z_2 + n_2r_2)$. Let PQ be the line of shortest distance, its direction ratios will be

$$[(l_1r_1 + x_1 - x_2 - l_2r_2), (m_1r_1 + y_1 - y_2 - m_2r_2), (n_1r_1 + z_1 - z_2 - n_2r_2)]$$

This line is perpendicular to both the given lines. By using condition of perpendicularity we obtain two equations in r_1 and r_2. So by solving these, values of r_1 and r_2 and subsequently point P and Q can be found. The distance PQ is the shortest distance.

Cartesian form : The shortest distance between two lines is given by

$$\frac{1}{\sqrt{\sum(l_1m_2-l_2m_1)^2}}\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}.$$

Vector form : The shortest distance between two lines $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ is $\left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$.

If any straight line is given in general form then it can be transformed into symmetrical form and we can further proceed.

SHORTEST DISTANCE BETWEEN TWO PARALLEL LINES

If two lines l_1 and l_2 are parallel, then they are coplanar. Let the lines be given by $\vec{r}=\vec{a}_1+\lambda\vec{b}$ and $\vec{r}=\vec{a}_2+\mu\vec{b}$, where $\vec{a}_1$ is the position vector of a point on l_1 and $\vec{a}_2$ is the position vector of a point on l_2.

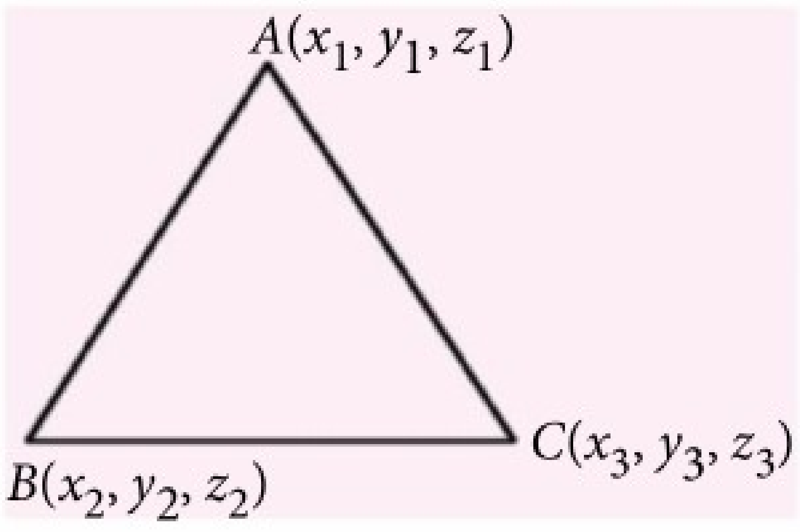
The shortest distance between the given parallel lines is $d=\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|$.

AREA OF A TRIANGLE

$$\Delta_x=\frac{1}{2}\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix}, \Delta_y=\frac{1}{2}\begin{vmatrix} x_1 & z_1 & 1 \\ x_2 & z_2 & 1 \\ x_3 & z_3 & 1 \end{vmatrix},$$

$$\Delta_z=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

So, area of ΔABC is given by the relation $\Delta^2=\Delta_x{}^2+\Delta_y{}^2+\Delta_z{}^2$



THE PLANE

Consider the locus of a point $P(x, y, z)$. If x, y, z are allowed to vary with any restriction for their different combinations, we have a set of points for P. The surface on which these points lie, is called the locus of P. It may be a plane or any curved surface. If Q be any other point on its locus and all points of the straight line PQ lie on it then it is a plane. In other words for the plane the straight line PQ, however small and in whatever direction will completely lie on the locus, otherwise it will be a curved surface.

EQUATION OF A PLANE IN NORMAL FORM, PASSING THROUGH A FIXED POINT

Cartesian form : General equation of a plane is $ax + by + cz + d = 0$. (where a, b, c gives the direction ratios of the normal to the plane).

Equation of the plane in normal form is $lx + my + nz = p$ where p is the length of the normal from the origin to the plane and (l, m, n) be the direction cosines of the normal .

Vector form : Equation of the plane in vector form, $\vec{r}\cdot\hat{n} = d$, where $\hat{n}$ is a unit vector perpendicular to the plane and d is length of perpendicular drawn from origin to the plane.

Equation of YZ plane is $x = 0$,

equation of plane parallel to YZ plane is $x = d$.

Equation of ZX plane is $y = 0$,

equation of plane parallel to ZX plane is $y = d$.

Equation of XY plane is $z = 0$,

equation of plane parallel to XY plane is $z = d$.

Four points namely $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$ will be coplanar if one point lies on the plane passing through other three points.

EQUATION OF A PLANE PERPENDICULAR TO A GIVEN LINE AND PASSING THROUGH A GIVEN POINT

Cartesian form : The equation to the plane passing through $P(x_1, y_1, z_1)$ and is perpendicular to given line having direction ratios (a, b, c) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$.

Vector form : The equation to the plane passing through a point whose position vector is $\vec{r}_1$ and unit vector perpendicular to the plane is $\hat{n}$, is $(\vec{r} - \vec{r}_1)\cdot\hat{n} = 0$.

EQUATION OF A PLANE PASSING THROUGH THREE NON COLLINEAR POINTS

The equation of the plane passing through three non-collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} (x-x_1) & (y-y_1) & (z-z_1) \\ (x_2-x_1) & (y_2-y_1) & (z_2-z_1) \\ (x_3-x_1) & (y_3-y_1) & (z_3-z_1) \end{vmatrix} = 0.$$

INTERCEPT FORM OF THE EQUATION OF A PLANE

Intercept form of the equation of a plane is given by $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ where a, b, c are x-intercept, y-intercept and z-intercept respectively.

ANGLE BETWEEN TWO PLANES

Cartesian form : Angle between the planes is defined as angle between normals to the planes drawn from any point. Angle between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is

$$\cos^{-1}\left(\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{\left(a^2_1+b^2_1+c^2_1\right)\left(a^2_2+b^2_2+c^2_2\right)}}\right).$$

Vector form : Angle between the planes is

$$\cos^{-1}\left(\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right).$$

where $\vec{n}_1$ and $\vec{n}_2$ are vectors perpendicular to the planes.

Note : (i) If $a_1a_2 + b_1b_2 + c_1c_2 = 0$, then the planes are perpendicular to each other.

(ii) If $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ then the planes are parallel to each other.

DISTANCE OF A POINT FROM A PLANE

Cartesian form : The length of the perpendicular of the point $P(x_1, y_1, z_1)$ from the plane $ax + by + cz + d = 0$ is $\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$.

Vector form : The length of the perpendicular from the point $P(\vec{r}_1)$ to the plane $\vec{r}.\hat{n} = d$ is $|\vec{r}_1.\hat{n} - d|$.

DISTANCE BETWEEN TWO PARALLEL PLANES

The distance between two parallel planes is the algebraic difference of perpendicular distances on the planes from origin.

Let two parallel planes be $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$.

Then distance between them is given by $\frac{|d_2-d_1|}{\sqrt{a^2+b^2+c^2}}$.

POSITION OF TWO POINTS W.R.T. A PLANE

Two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ on the same or opposite sides of a plane $ax + by + cz + d = 0$ according to $ax_1 + by_1 + cz_1 + d$ and $ax_2 + by_2 + cz_2 + d$ are of same or opposite signs. The plane divides the line joining the points P and Q externally or internally according to P and Q are lying on same or opposite sides of the plane.

EQUATION OF THE PLANE THROUGH A GIVEN LINE

- If equation of line is given in general form as $a_1x + b_1y + c_1z + d_1 = 0$, $a_2x + b_2y + c_2z + d_2 = 0$ then the equation of plane passing through this line is $(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$.

- If equation of the line is given in symmetrical form as $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$, then equation of plane is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$, where a, b, c are given by $al+bm+cn=0$.

PLANE PASSING THROUGH THE INTERSECTION OF TWO GIVEN PLANES

Equation of plane passing through the line of intersection of two planes $u=0$ and $v=0$ is $u+\lambda v=0$.

ANGLE BETWEEN A LINE AND A PLANE

If equation of a plane is $ax+by+cz+d=0$, then direction ratios of normal to this plane are a, b, c.

The equation of straight line is $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$, then angle between normal to the plane and straight line is given by $\sin\theta=\frac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$, where θ is the angle between the plane and the line.

The plane and the straight line will be parallel if $al+bm+cn=0$.

The plane and the straight line will be perpendicular if $\frac{a}{l}=\frac{b}{m}=\frac{c}{n}$.

EQUATION OF THE PLANES BISECTING THE ANGLES BETWEEN TWO GIVEN PLANES

- **Cartesian form :** Equation of two planes bisecting the angles between the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is given by

$$\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}=\pm\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}}$$

- **Vector form :** Let the two planes be $\vec{r}\cdot\vec{n}_1=d_1$ and $\vec{r}\cdot\vec{n}_2=d_2$ then equation of the planes bisecting the angles of given planes is given by

$$\frac{|\vec{r}\cdot\vec{n}_1-d_1|}{|\vec{n}_1|}=\frac{|\vec{r}\cdot\vec{n}_2-d_2|}{|\vec{n}_2|} \text{ or } \vec{r}\cdot(\vec{n}_1\pm\vec{n}_2)=\frac{d_1}{|\vec{n}_1|}\pm\frac{d_2}{|\vec{n}_2|}$$

PROBLEMS

Single Correct Answer Type

1. Let $a, b>0$ and $\vec{\alpha}=\frac{\hat{i}}{a}+\frac{4\hat{j}}{b}+b\hat{k}$ & $\vec{\beta}=b\hat{i}+a\hat{j}+\frac{1}{b}\hat{k}$, then the maximum value of $\frac{10}{5+\vec{\alpha}\cdot\vec{\beta}}$ is

(a) 1 (b) 2 (c) 4 (d) 8

2. Equation of a plane bisecting the angle between the planes $2x-y+2z+3=0$ and $3x-2y+6z+8=0$ is

(a) $5x-y-4z-45=0$

(b) $5x-y-4z-3=0$

(c) $23x+13y+32z-45=0$

(d) $23x-13y+32z+5=0$

3. If the angle θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is such that $\sin\theta=1/3$, then the value of λ is

(a) 3/4 (b) –4/3 (c) 5/3 (d) –3/5

4. Two adjacent sides of a parallelogram $ABCD$ are given by $\overrightarrow{AB}=2\hat{i}+10\hat{j}+11\hat{k}$ and $\overrightarrow{AD}=-\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB then the cosine of angle α is given by

(a) $\frac{8}{9}$ (b) $\frac{\sqrt{17}}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4\sqrt{5}}{9}$

5. Equation of the plane through (3, 4, –1) which is parallel to the plane $\vec{r}\cdot(2\hat{i}-3\hat{j}+5\hat{k})+7=0$ is

(a) $\vec{r}\cdot(2\hat{i}-3\hat{j}+5\hat{k})+11=0$

(b) $\vec{r}\cdot(3\hat{i}+4\hat{j}-\hat{k})+11=0$

(c) $\vec{r}\cdot(3\hat{i}+4\hat{j}-\hat{k})+7=0$

(d) $\vec{r}\cdot(2\hat{i}-3\hat{j}+5\hat{k})-7=0$

6. For unit vectors $\vec{b}$ and $\vec{c}$ and for any non zero vector $\vec{a}$, the value of $\{\{(\vec{a}+\vec{b})+(\vec{a}+\vec{c})\}\times(\vec{b}+\vec{c})\}\cdot(\vec{b}+\vec{c})$ is

(a) $|\vec{a}|^2$ (b) $2|\vec{a}|^2$ (c) $3|\vec{a}|^2$ (d) 0

7. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=-\hat{i}+\hat{j}+\hat{k}$, $\vec{c}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{d}=\hat{i}+\hat{j}-\hat{k}$. Then, the line of intersection of planes one determined by $\vec{a}$, $\vec{b}$ and other determined by $\vec{c}$, $\vec{d}$ is perpendicular to

(a) x-axis

(b) y-axis

(c) both x and y-axes

(d) both y and z-axes

8. If a line with direction ratios (2, 2, 1) intersects the line $\frac{x-7}{3}=\frac{y-5}{2}=\frac{z-3}{1}$ and $\frac{x-1}{2}=\frac{y+1}{4}=\frac{z+1}{3}$ at A and B respectively, then $AB=$

(a) $\sqrt{2}$ (b) 2 (c) $\sqrt{3}$ (d) 3

9. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

(a) $x + 2y - 2z = 0$ (b) $3x + 2y - 2z = 0$
(c) $x - 2y + z = 0$ (d) $5x + 2y - 4z = 0$

10. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors and $\vec{d}$ be a non-zero vector, which is perpendicular to $\vec{a}+\vec{b}+\vec{c}$. Now, if $\vec{d}=(\sin x)(\vec{a}\times\vec{b})+(\cos y)(\vec{b}\times\vec{c})+2(\vec{c}\times\vec{a})$, then minimum value of $x^2 + y^2$ is

(a) π^2 (b) $\frac{\pi^2}{2}$ (c) $\frac{\pi^2}{4}$ (d) $\frac{5\pi^2}{4}$

Multiple Correct Answer Type

11. A vector of magnitude 2 along a bisector of the angle between the two vectors $2\hat{i}-2\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}-2\hat{k}$ is

(a) $\frac{2}{\sqrt{10}}(3\hat{i}-\hat{k})$ (b) $\frac{1}{\sqrt{26}}(\hat{i}-4\hat{i}+3\hat{k})$

(c) $\frac{2}{\sqrt{26}}(\hat{i}-4\hat{i}+3\hat{k})$ (d) $\frac{2}{\sqrt{5}}(\hat{i}-3\hat{k})$

12. The plane $lx + my = 0$ is rotated about its line of intersection with the plane $z = 0$, through an angle α, then equation of plane in its new position may be

(a) $lx+my+z\sqrt{l^2+m^2}\tan\alpha=0$

(b) $lx+my-z\sqrt{l^2+m^2}\tan\alpha=0$

(c) data is not sufficient
(d) none of these

13. Let a plane passes through origin and is parallel to the line $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z+1}{-2}$ such that distance between plane and the line is 5/3. Then equation of the plane is

(a) $x - 2y + 2z = 0$ (b) $x - 2y - 2z = 0$
(c) $2x + 2y + z = 0$ (d) $x + y + z = 0$

14. If x and y are two unit vectors and ϕ is the angle between them, then $\frac{1}{2}|x-y|$ is equal to

(a) 0 (b) $\frac{1}{\sqrt{2}}\sqrt{1-\cos\phi}$

(c) $\left|\sin\frac{\phi}{2}\right|$ (d) $\left|\cos\frac{\phi}{2}\right|$

15. If the median through A of ΔABC having vertices $A \equiv (2, 3, 5)$, $B \equiv (-1, 3, 2)$ and $C \equiv (\lambda, 5, \mu)$ is equally inclined to the axes, then

(a) $\lambda = 7$ (b) $\mu = 10$ (c) $\lambda = 10$ (d) $\mu = 7$

16. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, then which of the following statement(s) is/are true ?

(a) $\vec{a}\times(\vec{b}\times\vec{c}), \vec{b}\times(\vec{c}\times\vec{a}), \vec{c}\times(\vec{a}\times\vec{b})$ form a right handed system.

(b) $\vec{c},(\vec{a}\times\vec{b})\times\vec{c}, \vec{a}\times\vec{b}$ form a right handed system.

(c) $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}<0$ if $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

(d) $\frac{(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c})}{(\vec{b}\times\vec{c})\cdot(\vec{a}\times\vec{c})}=-1$ if $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

17. The plane $x - 2y + 7z + 21 = 0$

(a) contains the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$.

(b) contains the point $(0, 7, -1)$.

(c) is perpendicular to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{7}$.

(d) is parallel to the plane $x - 2y + 7z = 0$.

18. If $\vec{a},\vec{b},\vec{c}$ are non-coplanar, non-zero vectors and $\vec{r}$ is any vector in space then $[\vec{b}\,\vec{c}\,\vec{r}]\vec{a}+[\vec{c}\,\vec{a}\,\vec{r}]\vec{b}+[\vec{a}\,\vec{b}\,\vec{r}]\vec{c}$ is equal to

(a) $3[\vec{a}\,\vec{b}\,\vec{c}]\vec{r}$ (b) $[\vec{a}\,\vec{b}\,\vec{c}]\vec{r}$
(c) $[\vec{b}\,\vec{c}\,\vec{a}]\vec{r}$ (d) $6[\vec{a}\,\vec{b}\,\vec{c}]\vec{r}$

19. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u}=\vec{a}-(\vec{a}\cdot\vec{b})\vec{b}$ and $\vec{v}=\vec{a}\times\vec{b}$, then $|\vec{v}|=$

(a) $|\vec{u}|$ (b) $|\vec{u}|+|\vec{u}\cdot\vec{a}|$
(c) $|\vec{u}|+|\vec{u}\cdot\vec{b}|$ (d) $|\vec{u}|+\vec{u}\cdot(\vec{a}+\vec{b})$

Comprehension Type

Paragraph for Question No. 20 to 21

Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are forming a right handed system, if $\vec{a}\times\vec{b}=\vec{c}, \vec{b}\times\vec{c}=\vec{a}, \vec{c}\times\vec{a}=\vec{b}$.

20. If $\vec{x}=\vec{a}+\vec{b}-\bar{c}$, $\vec{y}=-\vec{a}+\vec{b}-2\vec{c}, \vec{z}=-\vec{a}+2\vec{b}-\vec{c}$, then a unit vector normal to the vector $\vec{x}+\vec{y}$ and $\vec{y}+\vec{z}$ is

(a) $\vec{a}$ (b) $\vec{b}$
(c) $\vec{c}$ (d) none of these

21. Let $\vec{x}=\vec{a}+\vec{b}$, $\vec{y}=2\vec{a}-\vec{b}$, then the point of intersection of straight lines $\vec{r}\times\vec{x}=\vec{y}\times\vec{x}$ and $\vec{r}\times\vec{y}=\vec{x}\times\vec{y}$ is

(a) $2\vec{b}$ (b) $3\vec{b}$
(c) $3\vec{a}$ (d) none of these

Paragraph for Question No. 22 to 23

Consider the equations of planes

$$P_1 \equiv \vec{r}\cdot(\hat{i}+2\hat{j}+\hat{k})-3=0,\ P_2 \equiv \vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})-5=0$$

22. The equation of plane passing through the intersection of $P_1 = 0$, $P_2 = 0$ and through the point $A(1, 1, 1)$ is

(a) $\vec{r}\cdot(5\hat{i}-4\hat{j}+5\hat{k})=6$ (b) $\vec{r}\cdot(5\hat{i}+5\hat{j}-4\hat{k})=6$

(c) $\vec{r}\cdot(5\hat{i}+5\hat{j}+4\hat{k})=14$ (d) none of these

23. The line of intersection of planes $P_1 = 0$, $P_2 = 0$ is parallel to

(a) $3\hat{i}-5\hat{j}-\hat{k}$ (b) $3\hat{i}+\hat{j}-5\hat{k}$

(c) $2\hat{i}-\hat{j}-\hat{k}$ (d) none of these

Matrix-Match Type

24. Match the following :

Column-I		Column-II	
A.	The area of the triangle whose vertices are the points, with rectangular cartesian coordinates (1, 2, 3), (–2, 1, –4), (3, 4, –2) is	p.	0
B.	The value of $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})+(\vec{b}\times\vec{c})\cdot(\vec{a}\times d)+(\vec{c}\times\vec{a})\cdot(\vec{b}\times\vec{d})$ is	q.	$\frac{9}{2}$
C.	If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect then value of k is	r.	$\frac{\sqrt{1218}}{8}$
D.	$\vec{a}=2\hat{i}+3\hat{j}-\hat{k}, \vec{b}=-\hat{i}+2\hat{j}-4\hat{k}$, $\vec{c}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{d}=3\hat{i}+2\hat{j}+\hat{k}$ then $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=$	s.	21

25. Match the following :

Column-I		Column-II	
A.	The value of α for which the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}+\alpha\hat{k}$ and $3\hat{i}-4\hat{j}+5\hat{k}$ are coplanar is	p.	4
B.	The area of a parallelogram having adjacent sides $\vec{a}=3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}-3\hat{j}+4\hat{k}$ is	q.	–3
C.	If $\vec{r}\cdot\vec{a}=0, \vec{r}\cdot\vec{b}=0$ and $\vec{r}\cdot\vec{c}=0$ for some non-zero vector $\vec{r}$, then the value of $[\vec{a}\ \vec{b}\ \vec{c}]$ is	r.	$10\sqrt{3}$
D.	The volume of parallelepiped whose sides are given $\overrightarrow{OA}=2\hat{i}-3\hat{j}$, $\overrightarrow{OB}=\hat{i}+\hat{j}-\hat{k}$ and $\overrightarrow{OC}=3\hat{i}-\hat{k}$ is	s.	0

Numerical Answer Type

26. Let $\vec{c}$ be a unit vector coplanar with $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ such that $\vec{c}$ is perpendicular to $\vec{a}$. If p be the projection of $\vec{c}$ along $\vec{b}$, then evaluate $\frac{\sqrt{11}}{p}$.

27. Find the distance of the point $\hat{i}+2\hat{j}+3\hat{k}$ from the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=5$ measured parallel to the vector $2\hat{i}+3\hat{j}-6\hat{k}$.

28. If $\vec{a}, \vec{b}, \vec{c}$ are the three unit vectors and α, β, γ are scalars such that $\vec{c}=\alpha\vec{a}+\beta\vec{b}+\gamma(\vec{a}\times\vec{b})$. It is given that $\vec{a}\cdot\vec{b}=0$ and $\vec{c}$ makes equal angle with both $\vec{a}$ and $\vec{b}$ then evaluate $\alpha^2+\beta^2+\gamma^2$.

29. The equation of the plane passing through the intersection of the planes $2x - 5y + z = 3$ and $x + y + 4z = 5$ and parallel to the plane $x + 3y + 6z = 1$ is $x + 3y + 6z = k$, where k is

30. If $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$ and $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=\frac{1}{2}$ then the value of $4\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$ is

SOLUTIONS

1. (a) : $\vec{\alpha}\cdot\vec{\beta}=\frac{b}{a}+\frac{4a}{b}+1$ as $\frac{b}{a}+\frac{4a}{b}+1\geq 5$

So, $\left(\frac{10}{5+\vec{\alpha}\cdot\vec{\beta}}\right)_{\max}=1$

2. (b) : Equation of the plane bisecting the angle between the given planes are

$$\frac{2x-y+2z+3}{\sqrt{(2)^2+(-1)^2+2^2}}=\pm\frac{3x-2y+6z+8}{\sqrt{3^2+(-2)^2+6^2}}$$

$\Rightarrow$ $7(2x - y + 2z + 3) = \pm 3(3x - 2y + 6z + 8)$

$\Rightarrow$ $5x - y - 4z - 3 = 0$ taking the +ve sign, and

$23x - 13y + 32z + 45 = 0$ taking the – ve sign.

3. (c) : Since the line makes an angle θ with the plane. So, it makes an angle $\pi/2 - \theta$ with normal to the plane

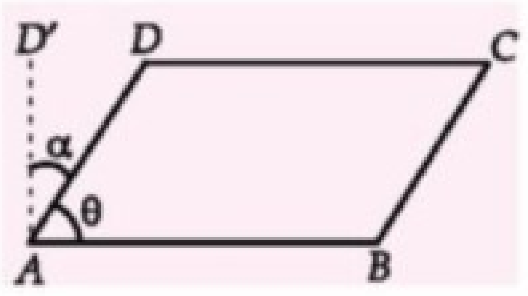
$$\therefore\ \cos\left(\frac{\pi}{2}-\theta\right)=\frac{2(1)+(-1)(2)+(\sqrt{\lambda})(2)}{\sqrt{1+4+4}\times\sqrt{4+1+\lambda}}$$

$$\Rightarrow\ \frac{1}{3}=\frac{2\sqrt{\lambda}}{3\sqrt{\lambda+5}}\Rightarrow\lambda+5=4\lambda\ \Rightarrow\ \lambda=5/3$$

4. (b) : Since, $\cos\theta=\frac{\overrightarrow{AB}\cdot\overrightarrow{AD}}{|\overrightarrow{AB}||\overrightarrow{AD}|}=\frac{8}{9}$

$\therefore\ \cos^{-1}\left(\frac{8}{9}\right)+\alpha=\frac{\pi}{2}$ (by hypothesis)

$\therefore \quad \sin\alpha = \dfrac{8}{9}$



$\Rightarrow \cos\alpha = \sqrt{1-\dfrac{64}{81}} = \dfrac{\sqrt{17}}{9}$

5. (a) : Equation of any plane parallel to the given plane is $\vec{r}\cdot(2\hat{i}-3\hat{j}+5\hat{k})+\lambda=0.$

Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, then equation of plane becomes $2x - 3y + 5z + \lambda = 0$

This plane passes through the point (3, 4, –1).

$\therefore \quad 2 \times 3 - 3 \times 4 + 5(-1) + \lambda = 0 \Rightarrow \lambda = 11$

Hence the equation of the required plane is

$\vec{r}\cdot(2\hat{i}-3\hat{j}+5\hat{k})+11=0$

6. (d)

7. (d) : $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a}\ \vec{b}\ \vec{d}]\vec{c}-[\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$

$= -4\vec{c}-4\vec{d} = -8\hat{i}$

Which is perpendicular to both y and z-axes.

8. (d) : Coordinates of A and B are $(7 + 3\alpha, 5 + 2\alpha, 3 + \alpha)$ and $(1 + 2\beta, - 1 + 4\beta, -1 + 3\beta)$ respectively.

d.r.'s of AB are (2, 2, 1)

$\therefore$ Equation of AB is

$$\frac{6+3\alpha-2\beta}{2}=\frac{3+\alpha-2\beta}{1}=\frac{4+\alpha-3\beta}{1}$$

$\Rightarrow \alpha = -2, \beta = 1$

$\therefore \quad A(1,1,1),\ B(3,3,2),\ AB = 3$

9. (c) : Vector along the required plane is

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 4 & 2\\ 4 & 2 & 3\end{vmatrix}=8\hat{i}-\hat{j}-10\hat{k}$$

So, normal vector $(\vec{n})$ to the plane is

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 8 & -1 & -10\\ 2 & 3 & 4\end{vmatrix}=26\hat{i}-52\hat{j}+26\hat{k}.$$

So, equation of the plane is $\vec{r}\cdot\vec{n}=0 \Rightarrow x-2y+z=0.$

10. (d) : Since $\vec{a},\vec{b},\vec{c}$ are non-coplanar $\Rightarrow [\vec{a},\vec{b},\vec{c}]\neq 0$

Also, $\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}$ are non-coplanar.

Given $\vec{d}=(\sin x)(\vec{a}\times\vec{b})+(\cos y)(\vec{b}\times\vec{c})+2(\vec{c}\times\vec{a})$.

Taking dot product with $\vec{a}+\vec{b}+\vec{c}$, we get

$0=\sin x\,[\vec{a}\ \vec{b}\ \vec{c}]+\cos y[\vec{a}\ \vec{b}\ \vec{c}]+2[\vec{a}\ \vec{b}\ \vec{c}]$

$\Rightarrow \sin x + \cos y + 2 = 0 \quad \Rightarrow \sin x + \cos y = -2$

$\Rightarrow x=(4n-1)\dfrac{\pi}{2};\quad y=(2n+1)\pi,\ n\in Z$

For least value of $x^2 + y^2$, $x=\dfrac{-\pi}{2},\ y=\pi$

and least value is $\dfrac{5\pi^2}{4}$.

11. (a, c) : $\vec{a}=2\hat{i}-2\hat{j}+\hat{k},\ \vec{b}=\hat{i}+2\hat{j}-2\hat{k}$

$\therefore \quad |\vec{a}| = 3,\ |\vec{b}| = 3$

$\therefore$ A vector along bisectors

$$=\frac{\vec{a}}{|\vec{a}|}\pm\frac{\vec{b}}{|\vec{b}|}=\hat{i}-\frac{1}{3}\hat{k},\ \frac{1}{3}\hat{i}-\frac{4}{3}\hat{j}+\hat{k}$$

$\therefore$ The required vector

$$=2.\frac{\hat{i}-\frac{1}{3}\hat{k}}{\sqrt{1^2+\left(-\frac{1}{3}\right)^2}},2.\frac{\frac{1}{3}\hat{i}-\frac{4}{3}\hat{j}+\hat{k}}{\sqrt{\left(\frac{1}{3}\right)^2+\left(-\frac{4}{3}\right)^2+1^2}}$$

$$=\frac{2}{\sqrt{10}}(3\hat{i}-\hat{k}),\ \frac{2}{\sqrt{26}}(\hat{i}-4\hat{j}+3\hat{k})$$

12. (a, b) : Let equation of required plane is

$lx + my + \lambda z = 0$ ...(1)

Angle between (1) and $lx + my = 0$ is α.

$$\Rightarrow \cos\alpha=\frac{l^2+m^2}{\sqrt{l^2+m^2}\sqrt{l^2+m^2+\lambda^2}}$$

$$\Rightarrow \cos^2\alpha=\frac{l^2+m^2}{l^2+m^2+\lambda^2} \Rightarrow \lambda=\pm\sqrt{l^2+m^2}\tan\alpha$$

Hence, equation of plane is

$lx+my\pm z\sqrt{l^2+m^2}\tan\alpha=0$

13. (a, c) : Let the equation of plane be

$lx + my + nz = 0$, where l, m, n be d.c.'s

$\Rightarrow l^2 + m^2 + n^2 = 1$ ...(i)

Given line is $\dfrac{x-1}{2}=\dfrac{y+3}{-1}=\dfrac{z+1}{-2}$

$\therefore \quad 2l - m - 2n = 0$ ...(ii)

Also, $\dfrac{l-3m-n}{\sqrt{l^2+m^2+n^2}}=\dfrac{5}{3} \Rightarrow l-3m-n=\dfrac{5}{3}$ ...(iii)

Solving (i), (ii) and (iii) we get equation of plane as

$x - 2y + 2z = 0$ or $2x + 2y + z = 0.$

14. (b, c) : $|x - y|^2 = |x|^2 + |y|^2 - 2xy = 1 + 1 - 2\cos\phi$

$= 2(1 - \cos\phi)$

$\Rightarrow |x - y| = \sqrt{2}\sqrt{1-\cos\phi}$

$$\Rightarrow \ \frac{1}{2}\left|x-y\right| = \frac{1}{\sqrt{2}}\sqrt{1-\cos\phi} = \left|\sin\frac{\phi}{2}\right|$$

15. (a, b) : Mid-point of $BC = \left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right)$

d.r.'s of median through A are

$$\left(\frac{\lambda-1}{2}-2, 4-3, \frac{\mu+2}{2}-5\right) = \left(\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}\right)$$

The median is equally inclined to axes, so the direction ratios must be equal, so

$$\frac{\lambda-5}{2} = 1 = \frac{\mu-8}{2} \Rightarrow \lambda = 7, \mu = 10$$

16. (b, c, d) : (a) $\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})=\vec{0}$

$\Rightarrow$ vectors are coplanar, so do not form RHS.

(b) $(\vec{a}\times\vec{b})\times\vec{c}$, $\vec{a}\times\vec{b}$, $\vec{c}$ in this order form RHS.

$\Rightarrow$ $\vec{c}, (\vec{a}\times\vec{b})\times\vec{c}, \vec{a}\times\vec{b}$ also form RHS as they are in same cyclic order.

(c) $\vec{a}+\vec{b}+\vec{c}=\vec{0} \ \Rightarrow \ (\vec{a}+\vec{b}+\vec{c})^2=0$

$\Rightarrow \ \vec{a}^2+\vec{b}^2+\vec{c}^2 = -2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$

Hence, $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}<0$

(d) $\vec{a}+\vec{b}+\vec{c}=\vec{0} \ \Rightarrow \ \vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

Using this we get the result.

17. (a, b, c, d) : Conditions for the plane $ax+by+cz+d=0$ to contain the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ are $a\alpha + b\beta + c\gamma + d = 0$ and $al + bm + cn = 0$.

Here the condition is satisfied for option (a). For option (c), normal to plane and given line have same d.r.'s.

∴ The line is perpendicular to the given plane.

For option (b), clearly the point satisfies the given plane.

The d.r.'s of normal to the plane in option (d) are (1, –2, 7) which are same as the d.r.'s of normal to the given plane.

∴ The plane given in option (d) is parallel to given plane.

18. (b, c)

19. (a, c) : Given $\vec{v}=\vec{a}\times\vec{b} \Rightarrow |\vec{v}|=|\vec{a}||\vec{b}|\sin\theta=\sin\theta$

$\vec{u}=\vec{a}-(\vec{a}\cdot\vec{b})\vec{b}=\vec{a}-\vec{b}\cos\theta$

$\Rightarrow |\vec{u}|^2=(\vec{a}-\vec{b}\cos\theta)^2=|\vec{a}|^2+|\vec{b}|^2\cos^2\theta-2\vec{a}\cdot\vec{b}\cos\theta$

$=1+\cos^2\theta-2\cos^2\theta=1-\cos^2\theta=\sin^2\theta \ \Rightarrow \ |\vec{u}|=|\vec{v}|$

Again $\vec{u}\cdot\vec{b}=\vec{a}\cdot\vec{b}-(\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{b})=0 \Rightarrow |\vec{u}\cdot\vec{b}|=0$

So, $|\vec{v}|=|\vec{u}|+|\vec{u}\cdot\vec{b}|$

20. (d) : $\vec{x}+\vec{y}=2\vec{b}-3\vec{c}$ and $\vec{y}+\vec{z}=-2\vec{a}+3\vec{b}-3\vec{c}$

$$\therefore \ (\vec{x}+\vec{y})\times(\vec{y}+\vec{z}) = \begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ 0 & 2 & -3 \\ -2 & 3 & -3 \end{vmatrix} = 3\vec{a}+6\vec{b}+4\vec{c}$$

∴ Required unit vector $= \frac{3\vec{a}+6\vec{b}+4\vec{c}}{\sqrt{61}}$

21. (c) : $\vec{r}\times\vec{x}=\vec{y}\times\vec{x} \ \Rightarrow \ (\vec{r}-\vec{y})\times\vec{x}=0 \ \Rightarrow \vec{r}=\vec{y}+\lambda\vec{x}$

$\vec{r}\times\vec{y}=\vec{x}\times\vec{y} \ \Rightarrow \ (\vec{r}-\vec{x})\times\vec{y}=0 \Rightarrow \ \vec{r}=\vec{x}+\mu\vec{y}$

$\Rightarrow \ \vec{y}+\lambda\vec{x}=\vec{x}+\mu\vec{y}$

$\Rightarrow \ (2\vec{a}-\vec{b})+\lambda(\vec{a}+\vec{b})=(\vec{a}+\vec{b})+\mu(2\vec{a}-\vec{b})$

$\Rightarrow$ $2 + \lambda = 1 + 2\mu$, $-1 + \lambda = 1 - \mu$

$\Rightarrow$ $\mu = 1$, $\lambda = 1$

∴ The point of intersection is $3\vec{a}$.

22. (c) : The required plane is

$x + 2y + z - 3 + k(2x - y + z - 5) = 0$

Since it passes through A (1, 1, 1).

∴ $k = 1/3$

∴ The equation of plane is $5x + 5y + 4z - 14 = 0$,

i.e., $\vec{r}\cdot(5\hat{i}+5\hat{j}+4\hat{k})=14$

23. (b) : The line of intersection of planes $\vec{r}\cdot\vec{n}_1=d_1$ and $\vec{r}\cdot\vec{n}_2=d_2$ is parallel to $\vec{n}_1\times\vec{n}_2$.

24. A → r; B → p; C → q; D → s

(A) $\overrightarrow{OA}=\hat{i}+2\hat{j}+3\hat{k}$, $\overrightarrow{OB}=-2\hat{i}+\hat{j}-4\hat{k}$,

$\overrightarrow{OC}=3\hat{i}+4\hat{j}-2\hat{k}$

$$\text{Area}=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{\sqrt{1218}}{2}$$

(B) $((\vec{a}\times\vec{b})\times\vec{c})\cdot\vec{d}+((\vec{b}\times\vec{c})\times\vec{a})\cdot\vec{d}+((\vec{c}\times\vec{a})\times\vec{b})\cdot\vec{d}=0$

(C) Since, the given lines intersect.

$\therefore \ (2r+1, 3r-1, 4r+1)=(s+3, 2s+k, s)$

$\Rightarrow \ r=\frac{-3}{2}, s=-5 \ \Rightarrow \ k=\frac{9}{2}$

(D) Here, $\vec{a}\times\vec{b}=-10\hat{i}+9\hat{j}+7\hat{k}$

and $\vec{c}\times\vec{d}=-\hat{i}+2\hat{j}-\hat{k}$

$\therefore \ (\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=10+18-7=21$

25. A → q; B → r; C → s; D → p

(A) Since, vectors are coplanar

$$\therefore \begin{vmatrix}2 & -1 & 1\\ 1 & 2 & \alpha\\ 3 & -4 & 5\end{vmatrix}=0 \Rightarrow \alpha = -3$$

(B) Area of parallelogram $= \left\|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & -2\\ 1 & -3 & 4\end{vmatrix}\right\| = 10\sqrt{3}$

(C) Since, $\vec{a}, \vec{b}, \vec{c}$ are coplanar. $\therefore\ [\vec{a}\ \vec{b}\ \vec{c}] = 0$

(D) Volume of parallelepiped $= \begin{vmatrix}2 & -3 & 0\\ 1 & 1 & -1\\ 3 & 0 & -1\end{vmatrix} = 4$

26. (6) : Let $\vec{c} = x\vec{a} + y\vec{b}$, where x, y are scalars.

$\Rightarrow\ \vec{c} = x(\hat{i} - \hat{j} + 2\hat{k}) + y(2\hat{i} - \hat{j} + \hat{k})$

$\Rightarrow\ \vec{c} = \hat{i}(x + 2y) + \hat{j}(-x - y) + \hat{k}(2x + y)$

But $\vec{c}\cdot\vec{a} = 0 \Rightarrow 6x + 5y = 0 \Rightarrow y = \frac{-6x}{5}$

So, $\vec{c} = \frac{-7x}{5}\hat{i} + \frac{x}{5}\hat{j} + \frac{4x}{5}\hat{k}$

We have, $\frac{49x^2 + x^2 + 16x^2}{25} = 1 \Rightarrow x^2 = \frac{25}{66}$

$\therefore\ \vec{c} = \pm\frac{5}{\sqrt{66}}\left(\frac{-7}{5}\hat{i} + \frac{1}{5}\hat{j} + \frac{4}{5}\hat{k}\right);\ p = \frac{|\vec{c}\cdot\vec{b}|}{|\vec{b}|} = \frac{\sqrt{11}}{6}$

So, $\frac{\sqrt{11}}{p} = 6$

27. (7) : The distance of the point 'a' from the plane $\vec{r}\cdot\vec{n} = q$ measured in the direction of the unit vector $\vec{b} = \frac{q - \vec{a}\cdot\vec{n}}{\hat{b}\cdot\vec{n}}$

Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k},\ \vec{n} = \hat{i} + \hat{j} + \hat{k}$ and $q = 5$

Also, $\hat{b} = \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{\sqrt{(2)^2 + (3)^2 + (-6)^2}} = \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}$

$\therefore$ The required distance

$$= \left|\frac{5 - (\hat{i} + 2\hat{j} + 3\hat{k})\cdot(\hat{i} + \hat{j} + \hat{k})}{\frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})\cdot(\hat{i} + \hat{j} + \hat{k})}\right| = \left|\frac{5 - (1 + 2 + 3)}{\frac{1}{7}(2 + 3 - 6)}\right| = 7$$

28. (1) : Let $\vec{a}\cdot\vec{c} = \vec{b}\cdot\vec{c} = \cos\theta$ and also

$\vec{c} = \alpha\vec{a} + \beta\vec{b} + \gamma(\vec{a}\times\vec{b})$

Taking dot product with $\vec{a}$ both sides, we get $\cos\theta = \alpha$

Taking dot product with $\vec{b}$ both sides, we get $\cos\theta = \beta$

Taking dot product with $\vec{c}$ both sides, we get

$1 = \alpha\cos\theta + \beta\cos\theta + \gamma[\vec{a}\ \vec{b}\ \vec{c}]$

But $\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2 = \begin{vmatrix}1 & 0 & \cos\theta\\ 0 & 1 & \cos\theta\\ \cos\theta & \cos\theta & 1\end{vmatrix} = 1 - 2\cos^2\theta$

So, $1 = \cos^2\theta + \cos^2\theta + \gamma\sqrt{1 - 2\cos^2\theta}$

$\Rightarrow\ \gamma = \sqrt{1 - 2\cos^2\theta}$

So, $\alpha^2 + \beta^2 + \gamma^2 = 1$

29. (7) : Equation of plane passing through the intersection of the planes $2x - 5y + z = 3$ and $x + y + 4z = 5$ is

$(2x - 5y + z - 3) + \lambda(x + y + 4z - 5) = 0$

$\Rightarrow (2 + \lambda)x + (-5 + \lambda)y + (1 + 4\lambda)z - 3 - 5\lambda = 0$ ....(i)

which is parallel to the plane $x + 3y + 6z = 1$.

Then, $\frac{2 + \lambda}{1} = \frac{-5 + \lambda}{3} = \frac{1 + 4\lambda}{6}\ \therefore \lambda = \frac{-11}{2}$

From (i), we have, $-\frac{7}{2}x - \frac{21}{2}y - 21z + \frac{49}{2} = 0$

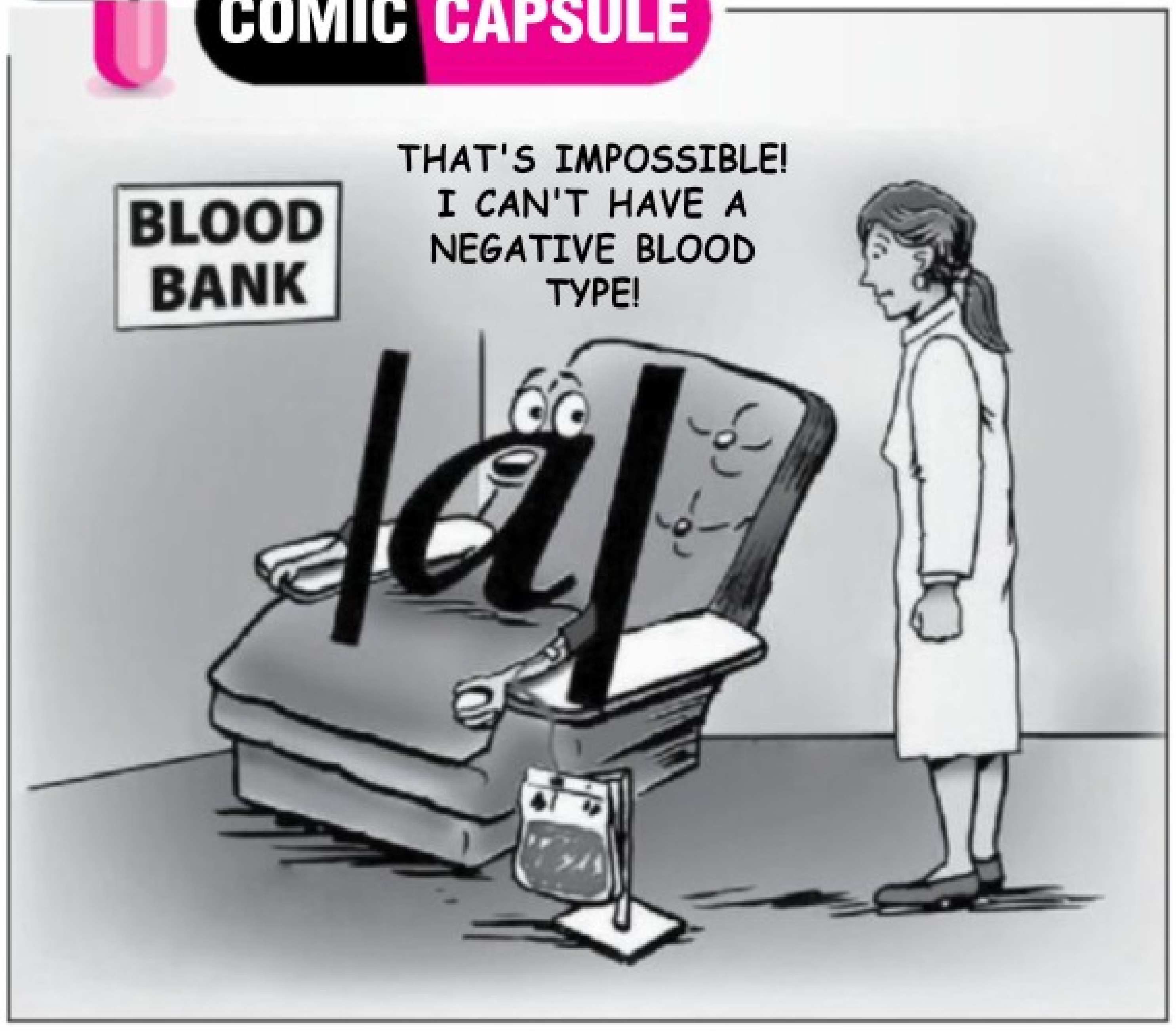
$\therefore\ x + 3y + 6z = 7$. Hence, $k = 7$

30. (2) : $[\vec{a}\ \vec{b}\ \vec{c}]^2 = [\vec{a}\ \vec{b}\ \vec{c}][\vec{a}\ \vec{b}\ \vec{c}]$

$$= \begin{vmatrix}\vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c}\\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c}\\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c}\end{vmatrix} = \begin{vmatrix}1 & 1/2 & 1/2\\ 1/2 & 1 & 1/2\\ 1/2 & 1/2 & 1\end{vmatrix} = \frac{1}{2}$$

$\Rightarrow\ 4[\vec{a}\ \vec{b}\ \vec{c}]^2 = 2$

COMIC CAPSULE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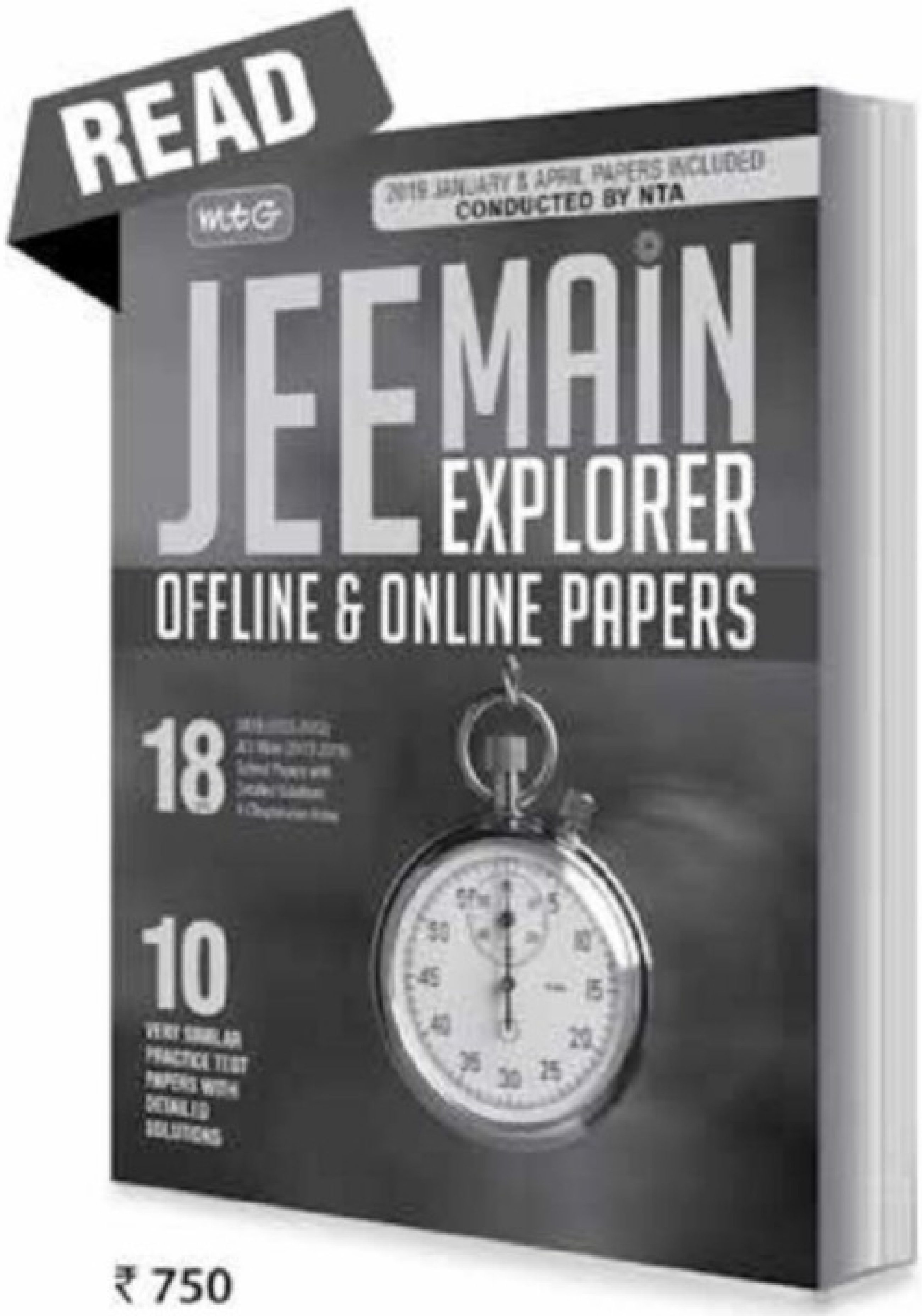
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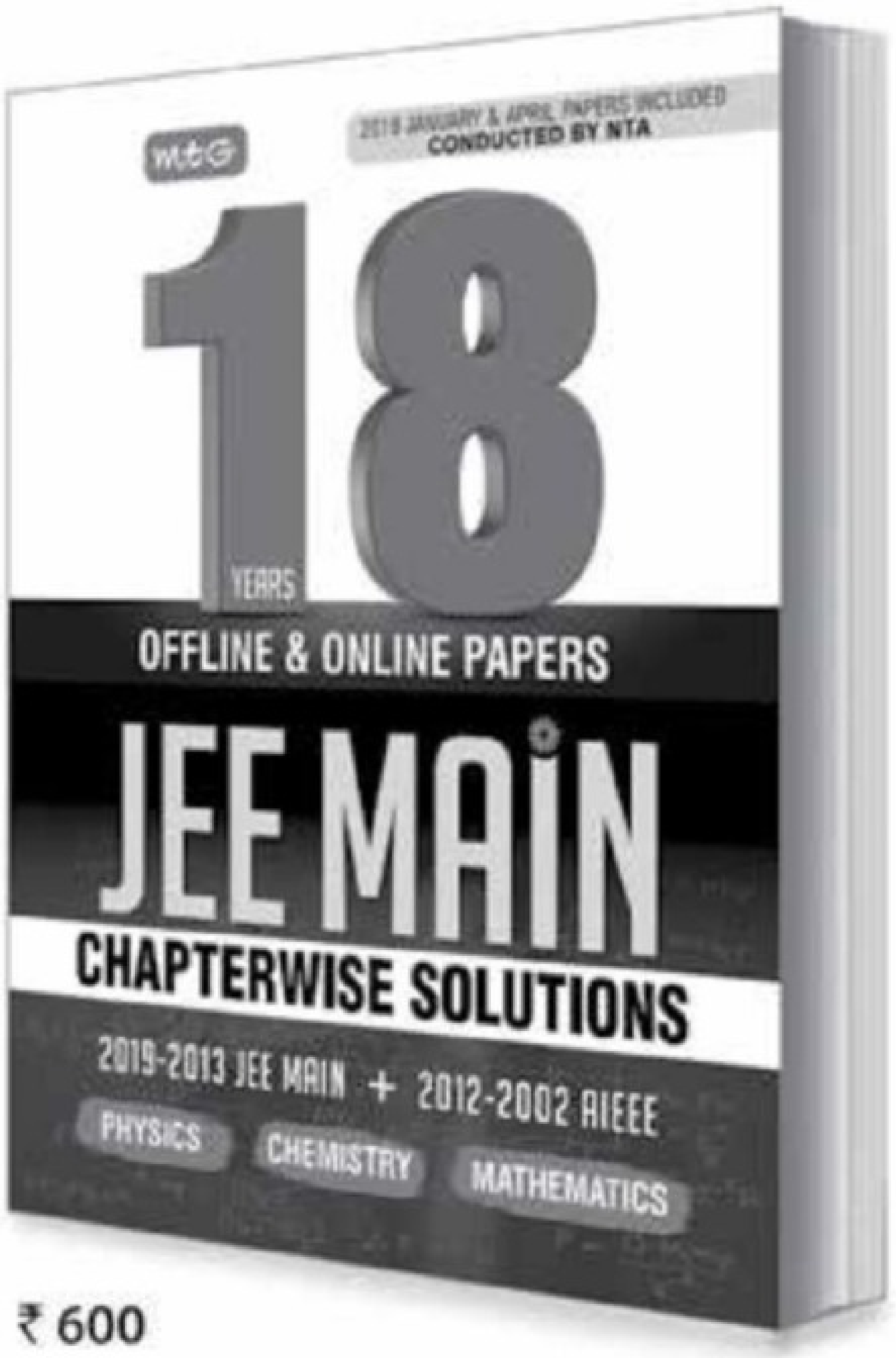
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MATHS MUSING

SOLUTION SET-201

1. (a) : We have, $\frac{xdy - ydx}{x^2} = 10x\sqrt{1 - \frac{y^2}{x^2}}\,dx$ ...(i)

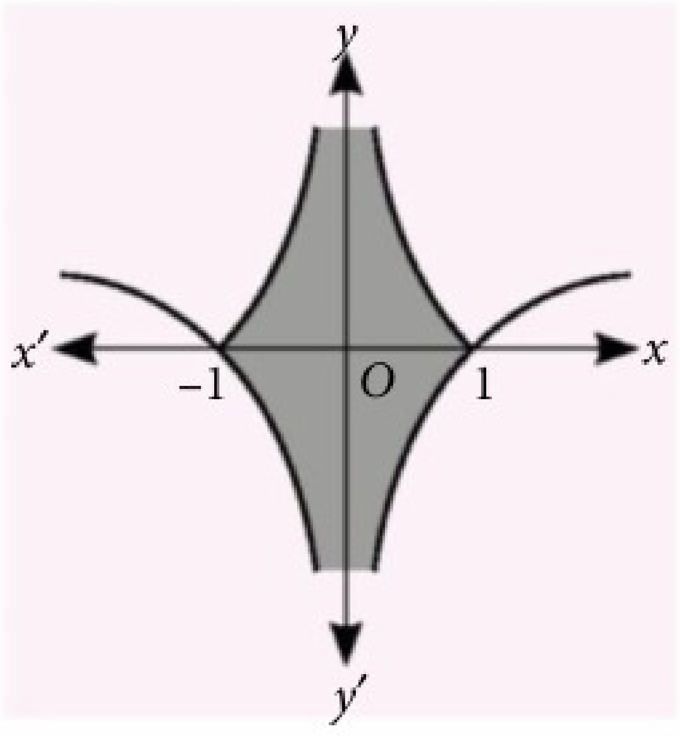
Put $\frac{y}{x} = t \Rightarrow \frac{xdy - ydx}{x^2} = dt$

$\therefore$ From (i), $\frac{dt}{\sqrt{1-t^2}} = 10x\,dx$

Integrating both sides, we get

$\sin^{-1} t = \frac{10x^2}{2} + C \Rightarrow y = x\sin(5x^2 + C)$

2. (a) : We know that, log x is defined for $x > 0$ and log $|x|$ is defined for all $x \in R - \{0\}$.
Also, $|\log x| \geq 0$ and $|\log|x|| \geq 0$



$\therefore$ Required area

$= 4\int_0^1 |\log x|\,dx = -4\int_0^1 \log x\,dx$

$= -4[x\log x - x]_0^1 = -4(-1) = 4$ sq.units

3. (b) : Number of parallelograms = ${}^5C_2 \times {}^4C_2 = 60$

4. (a) : $\because\ y = \frac{1}{3}[\sin x + [\sin x + [\sin x]]] = [\sin x]$

and $[y + [y]] = 2\cos x \Rightarrow [y] = \cos x$

Case I : When $-1 \leq \sin x < 0\ \therefore [\sin x] = -1 \Rightarrow y = -1$

$\therefore\ [-1] = \cos x \Rightarrow \cos x = -1$

$\Rightarrow\ \sin x = 0$ (impossible)

Case II : When $0 \leq \sin x < 1\ \therefore [\sin x] = 0 \Rightarrow y = 0$

$\therefore\ 0 = \cos x \Rightarrow \sin x = 1$ (impossible)

Case III : When $\sin x = 1 \Rightarrow y = 1$

$\therefore\ \cos x = 1 \Rightarrow \sin x = 0$ (impossible)

Hence, no solution.

5. (b) : Let the equation of the sphere be $x^2 + y^2 + z^2 - ax - by - cz = 0$.

This meets the axes at $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$.

Let (α, β, γ) be the coordinates of the centroid of the tetrahedron $OABC$. Then $\frac{a}{4} = \alpha, \frac{b}{4} = \beta, \frac{c}{4} = \gamma$

or $a = 4\alpha, b = 4\beta, c = 4\gamma$

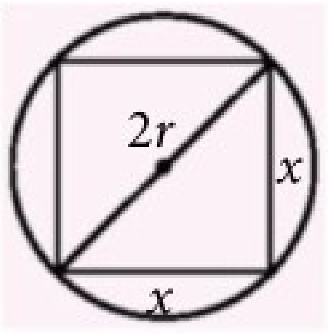
Now, radius of the sphere = $2k$

$\Rightarrow\ \frac{1}{2}\sqrt{a^2 + b^2 + c^2} = 2k \Rightarrow \alpha^2 + \beta^2 + \gamma^2 = k^2$

Hence, the locus of (α, β, γ) is $x^2 + y^2 + z^2 = k^2$

6. (c, d) : If r be the radius of the circle and x be the side of the square, then $x^2 + x^2 = (2r)^2 \Rightarrow x^2 = 2r^2$

Now, $p_1 = \frac{\text{Area of square}}{\text{Area of circle}} = \frac{x^2}{\pi r^2} = \frac{2r^2}{\pi r^2} = \frac{2}{\pi}$



$\therefore\ p_2 = 1 - p_1 = 1 - \frac{2}{\pi} = \frac{(\pi - 2)}{\pi}$

Now, $2 > \pi - 2 \Rightarrow \frac{2}{\pi} > \frac{\pi - 2}{\pi} \Rightarrow p_1 > p_2$

Also, $p_1^2 + p_2^2 = (p_1 + p_2)(p_1 - p_2) = 1\left(\frac{4 - \pi}{\pi}\right) = \frac{4}{\pi} - 1 < \frac{1}{3}$

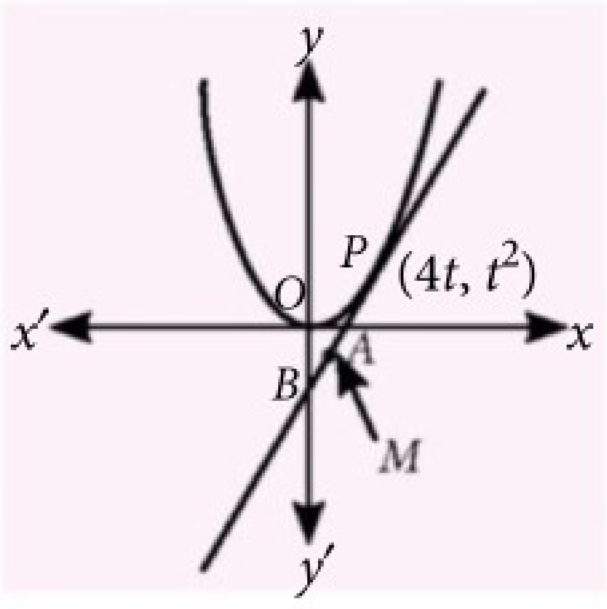
7. (b) : Any point on the hyperbola $xy = 16$ is $(4t, 4/t)$.
Normal at this point is $y - 4/t = t^2(x - 4t)$.
If the normal passes through $P(h, k)$, then
$k - 4/t = t^2(h - 4t) \Rightarrow 4t^4 - t^3h + tk - 4 = 0$
This equation has roots t_1, t_2, t_3, t_4 which are parameters of the four feet of normals on the hyperbola. Therefore,
$\Sigma t_1 = h/4,\ \Sigma t_1t_2 = 0,\ \Sigma t_1t_2t_3 = -k/4$ and $t_1t_2t_3t_4 = -1$

$\therefore\ \frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \frac{1}{t_4} = \frac{k}{4} \Rightarrow y_1 + y_2 + y_3 + y_4 = k$

Since, $t_1^2 + t_2^2 + t_3^2 + t_4^2 = k = \frac{h^2}{16}$

Hence, the locus of (h, k) is $x^2 = 16y$.

8. (c) : $x^2 = 16y$



The equation of tangent at P is

$x \cdot 4t = \frac{16(y + t^2)}{2}$

$\Rightarrow tx = 2y + 2t^2$

$A \equiv (2t, 0), B \equiv (0, -t^2)$

Let $M(h, k)$ is the mid-point of AB.

$\therefore\ h = t, k = -t^2/2 \Rightarrow 2k = -h^2$

Therefore, the locus of $M(h, k)$ is $x^2 + 2y = 0$.

9. (4) : We know that $|\text{adj } A^{-1}| = |A^{-1}|^2 = \frac{1}{|A|^2}$

$\Rightarrow\ |(\text{adj } A^{-1})^{-1}| = \frac{1}{|\text{adj } A^{-1}|} = |A|^2 = 2^2 = 4$

10. (a) : P $\rightarrow$ 4; Q $\rightarrow$ 3, R $\rightarrow$ 2; S $\rightarrow$ 1

(P) Here, $\vec{a} \times \vec{b} = 3\hat{i} - 3\hat{j} + 3\hat{k}$

Hence, the area of the triangle is $\frac{3\sqrt{3}}{2}$.

(Q) The area of the parallelogram is $3\sqrt{3}$.

(R) The area of the parallelogram whose diagonals are $2\vec{a}$ and $4\vec{b}$ is $\frac{1}{2}|2\vec{a} \times 4\vec{b}| = 12\sqrt{3}$.

(S) Volume of the parallelepiped
$= |(\vec{a} \times \vec{b}) \cdot \vec{c}| = |3 - 3 + 3| = 3$

◆ ◆

CBSE warm-up!

CLASS-XI

Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2019-20.

Series 6

Straight Lines

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) This question paper contains **36** questions.
(iii) Question **1**-**20** in Section-A are very short-answer-objective type questions carrying **1** mark each.
(iv) Question **21**-**26** in Section-B are short-answer type questions carrying **2** marks each.
(v) Question **27**-**32** in Section-C are long-answer-**I** type questions carrying **4** marks each.
(vi) Question **33**-**36** in Section-D are long-answer-**II** type questions carrying **6** marks each.

SECTION - A

1. If the slope of the line joining the points (3, 4) and (– 2, a) is equal to $-\frac{2}{5}$, then the value of a is equal to
(a) 6 (b) 4 (c) 3 (d) 2

2. Find the equation of line, which is parallel to y-axis and at a distance of 3 units from left of the origin is
(a) $x = 3$ (b) $y = -3$ (c) $x = -3$ (d) $y = 3$

3. Find the equation of the line through (–2, 3) with slope –4.
(a) $y - x + 2 = 0$ (b) $2x + 3y - 1 = 0$
(c) $3y + 4x + 5 = 0$ (d) $4x + y + 5 = 0$

4. The points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are vertices of a/an
(a) obtuse angled triangle (b) acute angled triangle
(c) right angled triangle (d) none of these

5. Find the slope of a line, which passes through the origin and the mid-point of the line segment joining the points (0, –4) and (8, 0).
(a) 1/2 (b) –1/2 (c) 1 (d) 2

6. Find the equation of the line where length of the perpendicular segment from the origin to the line is 4 and the inclination of the perpendicular segment with the positive direction of x-axis is 30°.
(a) $x-\sqrt{3}y=8$ (b) $x+\sqrt{3}y=8$
(c) $\sqrt{3}x+y=8$ (d) $\sqrt{3}x-y=8$

7. Find the equation of the line through the points (1, –1) and (3, 5).
(a) $2x - y = 3$ (b) $y = 2x - 4$
(c) $3x - y = 4$ (d) $y = 4x - 3$

8. The inclination of the line $x - y + 3 = 0$ with the positive direction of x-axis is
(a) 45° (b) 135° (c) –45° (d) –135°

9. Find the equation of the line perpendicular to the line $x - 7y + 5 = 0$ and having x-intercept 3.
(a) $7x + y + 21 = 0$ (b) $7x + y - 21 = 0$
(c) $7x - y - 21 = 0$ (d) None of these

10. The perpendicular distance from the point (1, – 1) to the line $x + 5y - 9 = 0$ is equal to

(a) $\sqrt{\frac{2}{13}}$ units (b) $\sqrt{\frac{13}{2}}$ units

(c) $\frac{13}{2}$ units (d) $\frac{2}{13}$ unit

11. Find the equation of the line passing through (–1, 1) and perpendicular to the line $2x + 3y + 4 = 0$.

12. The intercept cut off from y-axis is twice that from x-axis by the line and line is passes through (1, 2), then find its equation.

13. Find the equation of line whose mid point is (x_1, y_1) in between the axes.

14. If the slope of the line joining (2, 5) and (3, λ) is –2, find the value of λ.

15. Find the slope of the line joining the points (– 2, – 5) and (4, – 6).

16. Find the equation of the line perpendicular to x-axis and having intercept –2 on x-axis.

17. Convert the equation $3x + 4y - 24 = 0$ into intercept form.

18. Find the distance between the parallel lines $3x + 4y = 12$ and $3x + 4y = 2$.

19. Find the value of k for which the line $(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$ is parallel to the x-axis.

20. Find distance of point (3, –5) from the line $3x - 4y - 26 = 0$.

SECTION - B

21. The line through the points (h, 3) and (4, 1) intersects the line $7x - 9y - 19 = 0$ at right angle. Find the value of h.

22. A line through the point A(2, 0) which makes an angle of 30° with the positive direction of x-axis is rotated about A in clockwise direction through an angle 15°. Find the equation of the straight line in the new position.

OR

Find the equation of the line which is perpendicular to the line $x + y = 0$ and passes through (6, 4).

23. Find the equation of the perpendicular bisector of the line segment joining A(– 2, 3) and B(6, – 5).

24. The perpendicular distance of a line from the origin is 7 cm and its slope is –1. Find the equation of line.

OR

If the points (p + 1, 1), (2p + 1, 3) and (2p + 2, 2p) are collinear, then find the value of p.

25. Write the new equation of line $x + y = 2$, if the origin is shifted to (2, 2) by translation of axes.

26. Reduce the equation $\sqrt{3}x + y - 8 = 0$ into normal form. Find the length of perpendicular from the origin to the given line and angle made by the perpendicular with x-axis.

SECTION - C

27. If for a variable line $\frac{x}{a} + \frac{y}{b} = 1$, the condition $a^{-2} + b^{-2} = c^{-2}$ (c is a constant) is satisfied, find the locus of foot of the perpendicular drawn from the origin to this line.

28. Find the equation of the lines which cut off intercepts on the axes whose sum and product are 1 and – 6, respectively.

OR

Find angles between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$.

29. Find the equation of the straight line upon which the length of perpendicular from origin is $3\sqrt{2}$ units and this perpendicular makes an angle of 75° with the positive direction of x-axis.

30. The vertices of a triangle are A(10, 4), B(–4, 9) and C(–2, –1). Find the equation of the altitude through A. Also, find the foot of this altitude.

OR

Find the equations of the altitudes of the triangle whose vertices are A(7, – 1), B(–2, 8) and C(1, 2).

31. Find the equation of the internal bisector of angle BAC of the triangle ABC whose vertices A, B, C are (5, 2), (2, 3) and (6, 5) respectively.

32. Reduce $x + \sqrt{3}y + 4 = 0$ to the

(i) slope-intercept form and find its slope and y-intercept.

(ii) intercept form and find its intercepts on the axes.

SECTION - D

33. Find the equation of the line midway between the parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$.

34. Find the equations of the two lines which can be drawn through the point (2, 2) to make an angle of 45° with the line $x + y = 2$.

OR

A quadrilateral has the vertices at the points (–4, 2), (2, 6), (8, 5) and (9, –7). Show that the mid-points of the sides of this quadrilateral are vertices of a parallelogram.

35. Find the equation of the straight line which passes through the point of intersection of lines $3x - 4y - 7 = 0$ and $12x - 5y - 13 = 0$ and is perpendicular to the line $2x - 3y + 5 = 0$.

OR

Prove that product of the lengths of the perpendiculars drawn from the points $\left(\sqrt{a^2-b^2},0\right)$ and $\left(-\sqrt{a^2-b^2},0\right)$ to the line $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ is b^2.

36. Find the equation of the line passing through the point (2, 3) and making an intercept of length 3 units between the lines $y + 2x = 2$ and $y + 2x = 5$.

SOLUTIONS

1. **(a) :** Slope of the line = $\frac{-2}{5}$

$$\Rightarrow \frac{a-4}{-2-3}=\frac{-2}{5}\Rightarrow\frac{a-4}{-5}=\frac{-2}{5}\ \Rightarrow\ a-4=2\Rightarrow a=6$$

2. **(c) :** We know, the equation of line which is parallel to y-axis is of the form $x = k$.

Here, we take $k = -3$, because it is left side of the origin.

$\therefore\ \ x = -3$

3. **(d) :** Here $m = -4$ and given point (x_1, y_1) is (–2, 3).

By point slope form, equation of the line is

$$y - 3 = -4(x + 2)\ \Rightarrow\ 4x + y + 5 = 0$$

4. **(d) :** Let points be $A\left(0,\frac{8}{3}\right)$, $B(1, 3)$, and $C(82, 30)$.

Now, slope of line $AB=\frac{1}{3}$, slope of $BC=\frac{1}{3}$

As slope of AB = Slope of BC

$\therefore$ Points A, B and C are collinear.

5. **(b) :** Mid-point of the given line

$$=\left(\frac{0+8}{2},\frac{-4+0}{2}\right)=(4,-2)$$

Let m be the slope of the line containing the points (0, 0) and (4,–2), then

$$m=\frac{-2-0}{4-0}=\frac{-2}{4}=\frac{-1}{2}$$

6. **(c) :** The normal form of the equation of the line is $x\cos\alpha + y\sin\alpha = p$. Here $p = 4$ and $\alpha = 30°$. Therefore, the equation of the line is

$$x\cos 30° + y\sin 30° = 4$$

$$\Rightarrow\ x\frac{\sqrt{3}}{2}+y\frac{1}{2}=4\ \Rightarrow\ \sqrt{3}x+y=8$$

7. **(c) :** Here $x_1 = 1, y_1 = -1$ and $x_2 = 3, y_2 = 5$. Using two-point form, equation of the line is

$$y-(-1)=\frac{5-(-1)}{3-1}(x-1)\Rightarrow 3x-y-4=0$$

8. **(a) :** The equation of the line $x - y + 3 = 0$ can be written as $y = x + 3 \Rightarrow m = \tan\theta = 1$ and hence $\theta = 45°$.

9. **(b) :** Given equation of line is $x - 7y + 5 = 0$ ...(i)

Equation of line perpendicular to (i) is

$$7x + y + k = 0 \qquad ...(ii)$$

$\because$ Line (ii) passes through the point (3, 0).

$\therefore\ (7\times 3) + 0 + k = 0\ \Rightarrow\ k = -21$

Putting $k = -21$ in (ii), we get $7x + y - 21 = 0$

10. **(b) :** Required distance

$$=\left|\frac{1(1)+5(-1)-9}{\sqrt{1^2+5^2}}\right|=\left|\frac{-13}{\sqrt{26}}\right|=\sqrt{\frac{13}{2}}\text{ units}$$

11. The gradient of line $2x + 3y + 4 = 0$ is $-2/3$. Now the equation of line passing through (–1, 1) and having slope $m=-\frac{1}{-2/3}=\frac{3}{2}$ is $2(y - 1) = 3(x + 1)$.

12. Let the intercepts be a and $2a$, then the equation of line is $\frac{x}{a}+\frac{y}{2a}=1$, but it also passes through (1, 2), therefore $a = 2$.

Hence the required equation is $2x + y = 4$.

13. Intersection point on x-axis is $(2x_1, 0)$ and on y-axis is $(0, 2y_1)$. Thus equation of line passes through these points is $\frac{x}{x_1}+\frac{y}{y_1}=2$

14. Let $A(2, 5)$ and $B(3, \lambda)$.

Now, slope of $AB = -2\ \Rightarrow\ -2=\frac{\lambda-5}{3-2}$

$$\Rightarrow\ -2=\lambda-5\ \Rightarrow\ \lambda=5-2=3$$

15. Required slope $=\dfrac{-6-(-5)}{4-(-2)}=-\dfrac{1}{6}$

16. Since the required line is perpendicular to x-axis *i.e.*, parallel to y-axis, therefore, its equation will be $x = k$ ...(i)

Since the line has intercept -2 on x-axis, therefore it cuts x-axis at point $(-2, 0)$.

$\therefore$ Equation of required line is $x = -2$.

17. The given equation is $3x + 4y - 24 = 0$

Dividing both sides by 24, we get

$$\frac{3x}{24}+\frac{4y}{24}=1 \Rightarrow \frac{x}{8}+\frac{y}{6}=1$$

18. Given equations are $3x + 4y = 12$ and $3x + 4y = 2$

$\therefore$ Distance between them

$$=\left|\frac{12-2}{\sqrt{3^2+4^2}}\right|=\frac{10}{\sqrt{25}}=\frac{10}{5}=2 \text{ units}$$

19. Given equation of the line is

$(k-3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$

The given line is parallel to x-axis.

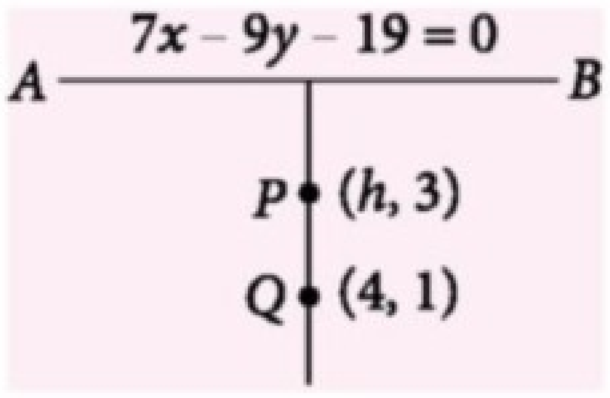
$\therefore$ Coeff. of $x = 0$

$\Rightarrow$ $k - 3 = 0 \Rightarrow k = 3$

20. Let d be the required distance, then

$$d=\frac{|3\times 3-4\times(-5)-26|}{\sqrt{(3)^2+(-4)^2}}=\frac{|9+20-26|}{\sqrt{9+16}}=\frac{3}{5} \text{ unit}$$

21. Slope of the line PQ passing through $P(h, 3)$ and $Q(4, 1)$ is $\dfrac{3-1}{h-4}=\dfrac{2}{h-4}$



Slope of the line AB,

$7x - 9y - 19 = 0$ is $\dfrac{7}{9}$.

The lines AB and PQ are perpendicular to each other.

$$\therefore \frac{2}{h-4}\times\frac{7}{9}=-1$$

$\Rightarrow$ $14 = -9(h - 4) \Rightarrow 9h = 36 - 14 = 22 \Rightarrow h=\dfrac{22}{9}$

22. Given, $A \equiv (2, 0)$, AB is the initial position of the line and AC is its new position.

Given, $\angle BAX = 30°$ and $\angle BAC = 15°$ $\therefore$ $\angle CAX = 15°$

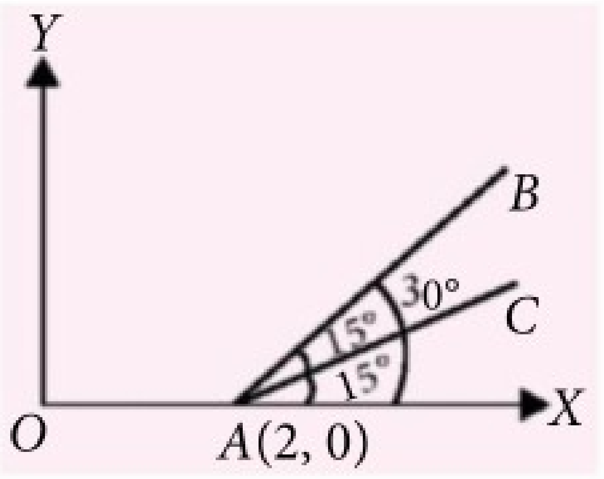
Now slope of line AC

$= \tan 15° = 2 - \sqrt{3}$

Now equation of line AC will be

$y - 0 = (2-\sqrt{3})(x-2)$

$\Rightarrow$ $(2-\sqrt{3})x-y-4+2\sqrt{3}=0$



OR

We have $x + y = 0 \Rightarrow y = -x$

$\therefore$ Slope $= -1$

and slope of a line perpendicular to the given line = 1

Now, the equation of the line with slope 1 and passing through the point (6, 4) is given by

$y - 4 = 1\,(x - 6)$

$\Rightarrow$ $x - y - 2 = 0$, which is the required equation.

23. Mid point of given line segment AB is

$\left(\dfrac{-2+6}{2},\dfrac{3-5}{2}\right)=\left(\dfrac{4}{2},\dfrac{-2}{2}\right)$ = (2, – 1).

Now, slope of the line segment AB is $\dfrac{-5-3}{6-(-2)}=\dfrac{-8}{8}=-1$

Slope of the required line segment is 1.

So equation of the required line is

$y + 1 = 1(x - 2) \Rightarrow x - y = 3$

24. Here, slope $m = \tan\theta = -1 \Rightarrow \theta = \tan^{-1}(-1)$

$\therefore$ $\theta = 135°$ and $p = 7$

Thus, the required line is $x \cos 135° + y \sin 135° = 7$

$\Rightarrow$ $x \cos(180° - 45°) + y \sin(180° - 45°) = 7$

$\Rightarrow$ $-x \cdot \cos 45° + y \sin 45° = 7$

$\Rightarrow$ $-x\cdot\dfrac{1}{\sqrt{2}}+y\cdot\dfrac{1}{\sqrt{2}}=7 \Rightarrow x-y+7\sqrt{2}=0$

OR

Let $A(x_1, y_1) = (p + 1, 1)$, $B(x_2, y_2) = (2p + 1, 3)$ and $C(x_3, y_3) = (2p + 2, 2p)$

Points A, B and C are collinear, if

Slope of AB = Slope of BC

i.e., $\dfrac{3-1}{(2p+1)-(p+1)}=\dfrac{2p-3}{(2p+2)-(2p+1)}$

$\Rightarrow$ $\dfrac{2}{p}=\dfrac{2p-3}{1} \Rightarrow 2p^2 - 3p - 2 = 0$

$\Rightarrow$ $(2p + 1)(p - 2) = 0 \Rightarrow p=\dfrac{-1}{2}, 2$

25. Here, $h = 2$ and $k = 2$

Let (x_1, y_1) be new co-ordinates of any point (x,y) on the given straight line.

$\therefore$ $x = x_1 + h \Rightarrow x = x_1 + 2$

and $y = y_1 + k \Rightarrow y = y_1 + 2$

Now, substituting the values in the given equation, $x + y = 2$ we have

$(x_1 + 2) + (y_1 + 2) - 2 = 0$

$\Rightarrow$ $x_1 + y_1 + 4 - 2 = 0 \Rightarrow x_1 + y_1 + 2 = 0$

Hence, the transformed equation of the line is $x + y + 2 = 0$

26. Given equation is $\sqrt{3}x+y-8=0$ ...(i)

Dividing (i) by $\sqrt{(\sqrt{3})^2+(1)^2}=2$, we get

$$\frac{\sqrt{3}}{2}x+\frac{1}{2}y=4 \Rightarrow \cos 30°\,(x)+\sin 30°\,(y)=4 \quad ...(ii)$$

Comparing (ii) with $x \cos \omega + y \sin \omega = p$, we get $p = 4$ and $\omega = 30°$

27. The given line is $\frac{x}{a}+\frac{y}{b}=1$ ...(i)

Equation of the line ⊥ to (i) from (0, 0) is $\frac{x}{b}-\frac{y}{a}=0$ ...(ii)

Let (α, β) be the point of intersection of (i) and (ii).

Then $\frac{\alpha}{a}+\frac{\beta}{b}=1$ ...(iii) and $\frac{\alpha}{b}-\frac{\beta}{a}=0$ ...(iv)

Squaring (iii) and (iv) and then adding, we get

$$\alpha^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)+\beta^2\left(\frac{1}{b^2}+\frac{1}{a^2}\right)=1$$

$$\Rightarrow (\alpha^2+\beta^2)\left(\frac{1}{a^2}+\frac{1}{b^2}\right)=1 \Rightarrow (\alpha^2+\beta^2)\left(\frac{1}{c^2}\right)=1$$

$\Rightarrow \alpha^2+\beta^2=c^2$.

Thus, locus of (α, β) is $x^2+y^2=c^2$.

28. Let a and b be the intercepts of the line on x and y axes respectively.

∴ Equation of the line will be $\frac{x}{a}+\frac{y}{b}=1$ ...(i)

Given, $a + b = 1$ ...(ii) and $ab = -6$ ...(iii)

Putting the value of b from (ii) in (iii), we get

$a(1-a)=-6 \Rightarrow a^2-a-6=0$

$\Rightarrow (a-3)(a+2) \Rightarrow a = 3, -2$

From (ii), when $a = 3$, $b = -2$ and when $a = -2$, $b = 3$

∴ Required equations of the lines are

$$\frac{x}{3}+\frac{y}{-2}=1 \text{ and } \frac{x}{-2}+\frac{y}{3}=1$$

$\Rightarrow 2x-3y-6=0$ and $3x-2y+6=0$

OR

Let the slopes of the given lines be m_1 and m_2 respectively.

∴ $y-\sqrt{3}x-5=0 \Rightarrow y=\sqrt{3}x+5$

and $\sqrt{3}y-x+6=0 \Rightarrow y=\frac{1}{\sqrt{3}}x-\frac{6}{\sqrt{3}}$

∴ $m_1=\sqrt{3}$ and $m_2=\frac{1}{\sqrt{3}}$

Let θ be the angle between the given lines.

Then, $\tan\theta=\left|\frac{m_2-m_1}{1+m_1m_2}\right|$

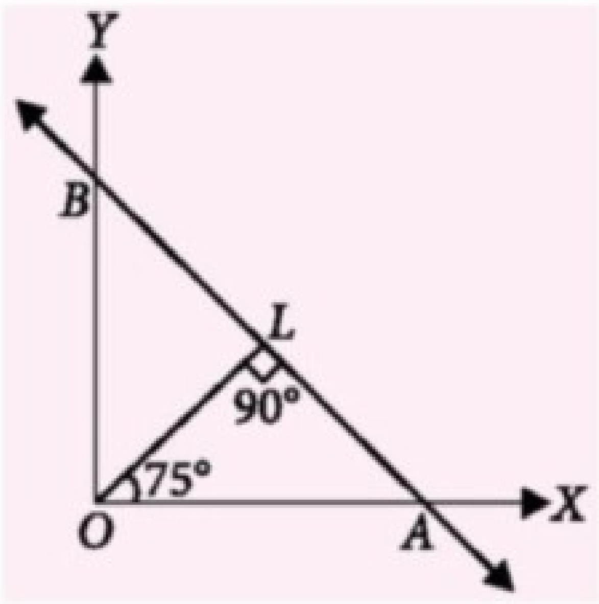
$$=\left|\frac{\frac{1}{\sqrt{3}}-\sqrt{3}}{1+(\sqrt{3})\left(\frac{1}{\sqrt{3}}\right)}\right|=\left|\frac{\left(\frac{1-3}{\sqrt{3}}\right)}{(1+1)}\right|=\pm\frac{2}{2\sqrt{3}}=\pm\frac{1}{\sqrt{3}}$$

$\Rightarrow \tan\theta=\frac{1}{\sqrt{3}}=\tan 30° \Rightarrow \theta=30°$

and $\tan\theta=\frac{-1}{\sqrt{3}}=\tan(180°-30°)=\tan 150°$

$\Rightarrow \theta=150°$

29. Let AB be the required line and OL be perpendicular to it.



Given, $OL=3\sqrt{2}$

and $\angle LOA=75°$

∴ Equation of line AB will be

$x\cos 75°+y\sin 75°=3\sqrt{2}$ ...(i)

Now, $\cos 75°=\cos(30°+45°)$

$=\frac{\sqrt{3}-1}{2\sqrt{2}}$

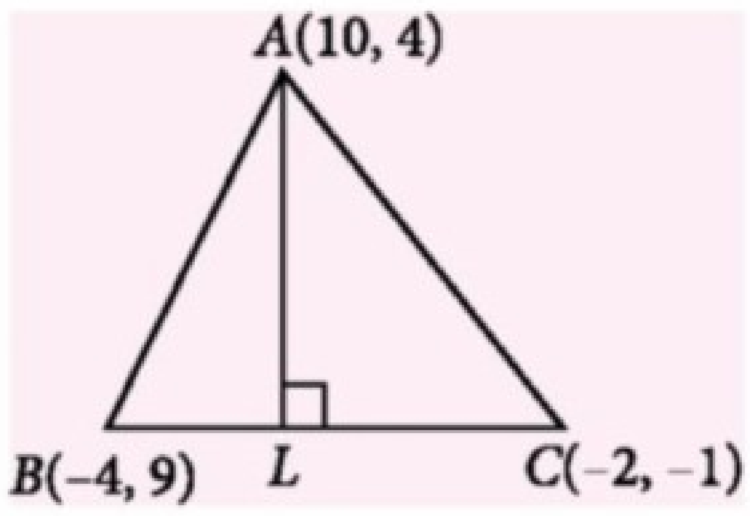
and similarly $\sin 75°=\frac{\sqrt{3}+1}{2\sqrt{2}}$

∴ From (i), equation of line AB is

$$x\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)+y\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)=3\sqrt{2}$$

$\Rightarrow (\sqrt{3}-1)x+(\sqrt{3}+1)y-12=0$

30. Altitude AL is the line through A and perpendicular to BC as shown.



Now, slope of BC

$=\frac{9-(-1)}{-4-(-2)}=\frac{10}{-2}=-5$

∴ Slope of $AL=1/5$

Equation of the altitude

AL is $y-4=\frac{1}{5}(x-10)$

$\Rightarrow 5y-20=x-10 \Rightarrow x-5y+10=0$ ...(i)

Now, equation of BC is

$y-9=-5(x+4) \Rightarrow 5x+y+11=0$ ...(ii)

Now, L is the point of intersection of BC and AL. Solving (i) and (ii) simultaneously for x and y, we get

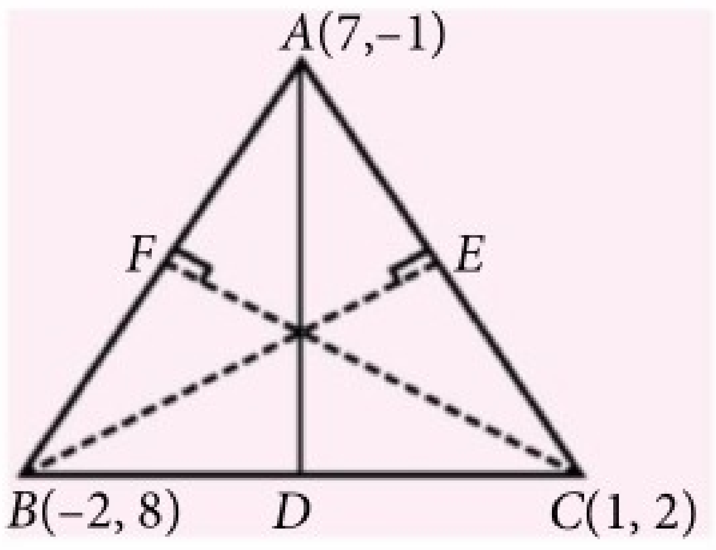
$$\frac{x}{-55-10}=\frac{y}{50-11}=\frac{1}{1+25} \Rightarrow x=-\frac{5}{2},\ y=\frac{3}{2}$$

∴ Foot of perpendicular, $L \equiv \left(-\frac{5}{2}, \frac{3}{2}\right)$

Monthly Test Drive-5 CLASS XII ANSWER KEY

1.	(d)	**2.**	(c)	**3.**	(c)	**4.**	(b)	**5.**	(b)
6.	(b)	**7.**	(c, d)	**8.**	(a, b, c)	**9.**	(a)	**10.**	(a, d)
11.	(a, d)	**12.**	(a, b, c)	**13.**	(a, d)	**14.**	(b)	**15.**	(b)
16.	(b)	**17.**	(5)	**18.**	(7)	**19.**	(6)	**20.**	(1)

OR



Let AD, BE and CF be three altitudes of triangle ABC. Let m_1, m_2 and m_3 be the slopes of AD, BE and CF respectively. Then, $AD \perp BC$

$\Rightarrow$ Slope of $AD \times$ Slope of $BC = -1$

$$\Rightarrow \ m_1 \times \left(\frac{2-8}{1+2}\right) = -1 \Rightarrow m_1 = \frac{1}{2}$$

$BE \perp AC \Rightarrow$ Slope of $BE \times$ Slope of $AC = -1$

$$\Rightarrow \ m_2 \times \left(\frac{-1-2}{7-1}\right) = -1 \Rightarrow m_2 = 2$$

and, $CF \perp AB \Rightarrow$ Slope of $CF \times$ Slope of $AB = -1$

$$\Rightarrow \ m_3 \times \frac{-1-8}{7+2} = -1 \Rightarrow m_3 = 1$$

Since, AD passes through $A(7, -1)$ and has slope $m_1 = \frac{1}{2}$.

So, its equation is

$$y + 1 = \frac{1}{2}(x - 7) \Rightarrow x - 2y - 9 = 0$$

Similarly, equation of BE is

$$y - 8 = 2(x + 2) \Rightarrow 2x - y + 12 = 0$$

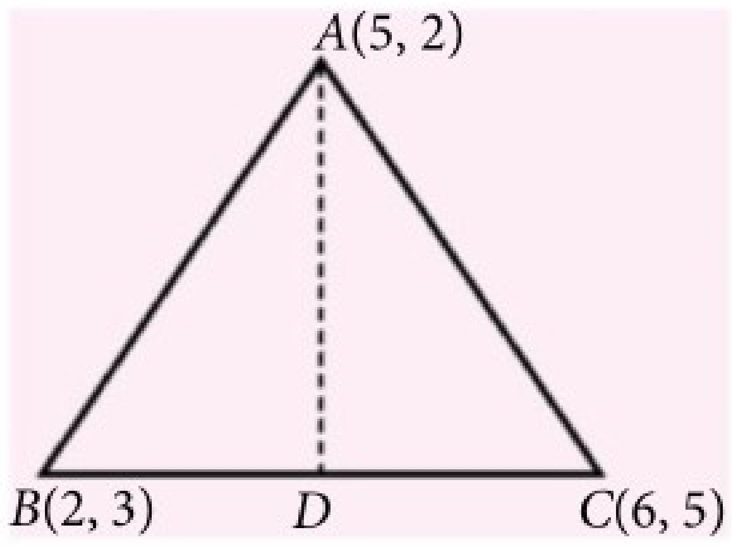
And equation of CF is $y - 2 = 1(x - 1) \Rightarrow x - y + 1 = 0$

31. We have, $AB = \sqrt{(5-2)^2 + (2-3)^2} = \sqrt{10}$

and, $AC = \sqrt{(5-6)^2 + (2-5)^2} = \sqrt{10}$

$\therefore \ AB : AC = \sqrt{10} : \sqrt{10} = 1 : 1$

The internal bisector AD of $\angle BAC$ divides BC in the ratio $AB : AC$ *i.e.*, 1 : 1.



So, coordinates of D are

$$\left(\frac{2+6}{2}, \frac{3+5}{2}\right) = (4, 4)$$

Equation of AD is

$$y - 2 = \frac{4-2}{4-5}(x - 5)$$

$\Rightarrow \ 2x + y - 12 = 0$

32. *Refer to answer 55, Page no. 223 of MTG CBSE Champion Mathematics, Class-11*

33. Converting each of the given equations to the form $y = mx + C$, we get

$$9x + 6y - 7 = 0 \ \Rightarrow \ y = \frac{-3}{2}x + \frac{7}{6} \qquad \text{...(i)}$$

$$3x + 2y + 6 = 0 \ \Rightarrow \ y = \frac{-3}{2}x - 3 \qquad \text{...(ii)}$$

Clearly, the slope of each one of the given lines is $\frac{-3}{2}$.

Let the given lines be $y = mx + C_1$ and $y = mx + C_2$.

Then, $m = \frac{-3}{2}$, $C_1 = \frac{7}{6}$ and $C_2 = -3$.

Let L be the required line. Then, L is parallel to each one of (i) and (ii), and equidistant from each one of them.

$\therefore$ Slope of $L = \frac{-3}{2}$.

Let the equation of L be $y = \frac{-3}{2}x + C$ ...(iii)

Then, distance between (i) and (iii) must be equal to the distance between (ii) and (iii).

$$\therefore \ \frac{|C_1 - C|}{\sqrt{1+m^2}} = \frac{|C_2 - C|}{\sqrt{1+m^2}} \Rightarrow |C_1 - C| = |C_2 - C|$$

$$\Rightarrow \ \left|\frac{7}{6} - C\right| = |-3 - C| \ \Rightarrow \ \left|\frac{7}{6} - C\right| = |3 + C|$$

$$\Rightarrow \ \frac{7}{6} - C = 3 + C \ \Rightarrow \ 2C = \frac{-11}{6} \ \Rightarrow \ C = \frac{-11}{12}$$

$\therefore$ Equation of L is $y = \frac{-3}{2}x - \frac{11}{12}$

i.e., $18x + 12y + 11 = 0$.

Hence, the line $18x + 12y + 11 = 0$ is midway between the parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$.

34. The given line is $x + y = 2$ ...(i)

Slope of this line = –1

Let the slope of the line through (2, 2), which makes an angle of 45° with line (i), be m, then

$$\tan 45^\circ = \left|\frac{m-(-1)}{1+m(-1)}\right| \Rightarrow \pm 1 = \frac{m+1}{1-m}$$

$\Rightarrow \ \pm(1 - m) = m + 1$

Case I : $1 - m = m + 1 \Rightarrow 2m = 0 \Rightarrow m = 0$

Hence, one of the required lines through (2, 2) is $y - 2 = 0(x - 2)$ (taking $m = 0$)

$\Rightarrow \ y = 2$

Case II : $-(1 - m) = m + 1 \Rightarrow -1 + m = m + 1$

$\therefore$ Slope is not defined.

$\therefore$ The other required line ought to be vertical.

$\Rightarrow$ The equation of this line is $x - 2 = 0$

($\because$ It passes through the point (2, 2))

OR

Refer to answer 19, Page no. 218 of MTG CBSE Champion Mathematics, Class-11

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35. Given lines are

$$3x - 4y - 7 = 0 \qquad ...(i)$$

$$12x - 5y - 13 = 0 \qquad ...(ii)$$

$$2x - 3y + 5 = 0 \qquad ...(iii)$$

Equation of any line through the point of intersection of lines (i) and (ii) is

$$3x - 4y - 7 + k(12x - 5y - 13) = 0$$

$$\Rightarrow \quad (3 + 12k)\, x - (4 + 5k)\, y - (7 + 13k) = 0 \qquad ...(iv)$$

Slope of line (iv) is $\frac{3+12k}{4+5k}$ and slope of line (iii) is $\frac{2}{3}$.

If line (iv) is perpendicular to line (iii), then

$$\left(\frac{3+12k}{4+5k}\right)\cdot\frac{2}{3}=-1 \Rightarrow 6 + 24k = -12 - 15k$$

$$\Rightarrow \quad 39k = -18 \quad \Rightarrow \quad k = -\frac{6}{13}$$

Putting the value of k in (iv), the equation of required line is $\left(3-\frac{72}{13}\right)x-\left(4-\frac{30}{13}\right)y-(7-6)=0$

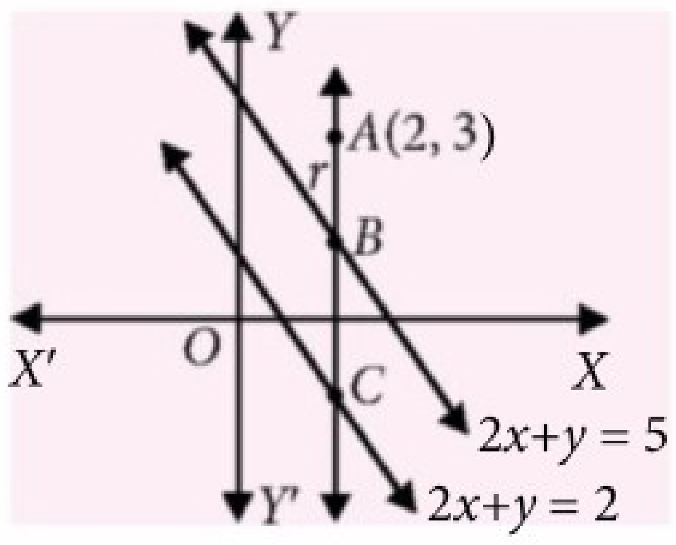
$$\Rightarrow \quad 33x + 22y + 13 = 0$$

OR

Refer to answer 93, Page no. 229 of MTG CBSE Champion Mathematics, Class - 11

36. The equation of the given lines are

$$2x + y = 2 \quad ...(i) \quad \text{and} \quad 2x + y = 5 \qquad ...(ii)$$



We observe that the lines given by equations (i) and (ii) are parallel. Suppose a line passing through $A(2, 3)$ and intercepting length $BC = 3$ between lines (i) and (ii) makes an angle θ with x-axis. The equation of this line in distance form is

$$\frac{x-2}{\cos\theta}=\frac{y-3}{\sin\theta} \qquad ..(iii)$$

Let $AB = r$. Then, $AC = AB + BC = (r + 3)$. Clearly, B and C are points on line (iii) at distance r and $r + 3$ respectively from A.

So, the coordinates of B and C are given by

$$\frac{x-2}{\cos\theta}=\frac{y-3}{\sin\theta}=r \text{ and } \frac{x-2}{\cos\theta}=\frac{y-3}{\sin\theta}=r+3 \text{ respectively.}$$

So, the coordinates of B and C are

$(2 + r\cos\theta, 3 + r\sin\theta)$ and

$(2 + (r + 3)\cos\theta, 3 + (r + 3)\sin\theta)$ respectively.

We observe that points B and C lie on lines (ii) and (i) respectively.

$$\therefore \quad 2(2 + r\cos\theta) + (3 + r\sin\theta) = 5$$

and $2\{2 + (r + 3)\cos\theta\} + \{3 + (r + 3)\sin\theta\} = 2$

$$\Rightarrow \quad r(2\cos\theta + \sin\theta) = -2 \text{ and } (r + 3)(2\cos\theta + \sin\theta) = -5$$

$$\Rightarrow \quad r=\frac{-2}{2\cos\theta+\sin\theta} \text{ and } r+3=\frac{-5}{2\cos\theta+\sin\theta}$$

$$\Rightarrow \quad \frac{-2}{2\cos\theta+\sin\theta}+3=\frac{-5}{2\cos\theta+\sin\theta} \quad [\text{On eliminating } r]$$

$$\Rightarrow \quad 3=\frac{-3}{2\cos\theta+\sin\theta} \qquad ...(iv)$$

$$\Rightarrow \quad 2\cos\theta + \sin\theta = -1 \quad \Rightarrow \quad 2\cos\theta = -1 - \sin\theta \qquad ...(v)$$

Squaring on both sides, we get

$$4\cos^2\theta = 1 + \sin^2\theta + 2\sin\theta$$

$$\Rightarrow \quad 4(1 - \sin^2\theta) = 1 + \sin^2\theta + 2\sin\theta$$

$$\Rightarrow \quad 5\sin^2\theta + 2\sin\theta - 3 = 0$$

$$\Rightarrow \quad (\sin\theta + 1)(5\sin\theta - 3) = 0$$

$$\Rightarrow \quad \sin\theta = -1 \text{ or } \sin\theta = 3/5$$

Putting $\sin\theta = -1$ in (v), we obtain $\cos\theta = 0$. Putting $\sin\theta = 3/5$ in (v), we obtain $\cos\theta = -4/5$. Substituting the values of $\sin\theta$ $\cos\theta$ in (iii), we obtain that the equations of the required lines are

$$\frac{x-2}{0}=\frac{y-3}{-1} \text{ and } \frac{x-2}{-4/5}=\frac{y-3}{3/5}$$

$$\Rightarrow \quad x - 2 = 0 \text{ and } 3x + 4y = 18$$

◆

COMIC CAPSULE

3.1 4 15 9 2
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MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 5 : Binomial Theorem

Time Taken : 60 Min.

Only One Option Correct Type

1. The expression $({}^{10}C_0)^2 - ({}^{10}C_1)^2 + ({}^{10}C_2)^2 - \dots + ({}^{10}C_8)^2 - ({}^{10}C_9)^2 + ({}^{10}C_{10})^2$ equals

(a) $10!$ (b) $({}^{10}C_5)^2$ (c) $-{}^{10}C_5$ (d) ${}^{10}C_5$

2. If the sum of the coefficients of various terms in the expansion of $(a + b)^n$ is 4096, then greatest coefficient in the expansion is

(a) 1594 (b) 792

(c) 924 (d) 2924

3. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of 5th and 6th terms is zero, then a/b equals

(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$ (c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$

4. If O be the sum of odd terms and E that of even terms in the expansion of $(x + a)^n$, then $O^2 - E^2 =$

(a) $(x^2 + a^2)^n$ (b) $(x^2 - a^2)^n$

(c) $(x - a)^{2n}$ (d) none of these

5. The coefficient of the term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$, is

(a) $\frac{1}{3}$ (b) $\frac{19}{54}$ (c) $\frac{17}{54}$ (d) $\frac{1}{4}$

6. If $C_0, C_1, C_2, \dots, C_n$ denote the coefficients in the expansion of $(1 + x)^n$, then

$C_0 + 3\cdot C_1 + 5\cdot C_2 + \dots + (2n + 1)C_n =$

(a) $n\cdot 2^n$ (b) $(n - 1)2^n$

(c) $(n + 1)2^{n-1}$ (d) $(n + 1)2^n$

One or More Than One Option(s) Correct Type

7. If the term independent of x in the expansion of $(\sqrt{x} - k/x^2)^{10}$ is 405, then value of k is

(a) 3 (b) -3 (c) 9 (d) -9

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8. If $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \dots + a_{12}x^{12}$, then
(a) $a_2 + a_4 + a_6 + \dots\dots + a_{12} = 31$
(b) $a_1 + a_3 + a_5 + \dots\dots + a_{11} = -32$
(c) $a_1 + a_2 + a_3 + \dots\dots + a_{12} = -1$
(d) none of these

9. If $(1 + 2x + 3x^2)^{10} = a_0 + a_1x + a_2x^2 + \dots + a_{20}x^{20}$, then
(a) $a_1 = 20$ (b) $a_2 = 210$
(c) $a_3 = 1500$ (d) none of these

10. Which of the following is an even positive integer?
(a) $(\sqrt{3}+1)^{2n+1} + (\sqrt{3}-1)^{2n+1}$
(b) $(\sqrt{3}+1)^{2n} + (\sqrt{3}-1)^{2n}$
(c) $(\sqrt{3}+1)^{2n+1} - (\sqrt{3}-1)^{2n+1}$
(d) none of these

11. In the expansion of $\left(\sqrt[3]{4} + \frac{1}{\sqrt[4]{6}}\right)^{20}$
(a) the number of rational terms = 4
(b) the number of irrational terms = 19
(c) the middle term is irrational
(d) the number of irrational terms = 17

12. In the expansion of $(x^2 + 2x + 2)^n$, (where n is a positive integer) then coefficient of
(a) x is $2^n \cdot n$ (b) x^2 is $n^2 \cdot 2^{n-1}$
(c) x^3 is $2^n \cdot {}^{n+1}C_3$ (d) none of these

13. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, then
(a) $C_1 + 2C_2 + 3C_3 + \dots + nC_n = n2^{n-1}$
(b) $C_0 + 2C_1 + 3C_2 + \dots + (n + 1)\, C_n = 2^{n-1}\,(n + 2)$
(c) $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$
(d) none of these

Comprehension Type

Let $\sum_{r=0}^{16} C_r x^r = (1+x)^{16}$. Then

14. $C_0 + C_4 + C_8 + C_{12} + C_{16} =$
(a) $2^8\,(2^8 - 1)$ (b) $2^8\,(2^8 + 1)$
(c) $2^7\,(2^7 - 1)$ (d) $2^7\,(2^7 + 1)$

15. $C_0 + C_3 + C_6 + C_9 + C_{12} + C_{15} =$
(a) $\frac{1}{3}(2^{16}+1)$ (b) $\frac{1}{3}(2^{16}-1)$
(c) $\frac{1}{3}(2^{15}+1)$ (d) $\frac{1}{3}(2^{15}-1)$

Matrix Match Type

16. Match the following columns:

	Column-I	Column-II	
P.	If ${}^{(n+1)}C_4 + {}^{(n+1)}C_3 + {}^{n+2}C_3 > {}^{(n+3)}C_3$, then possible value/values of n is/are	1.	4
Q.	The remainder when $(3053)^{456} - (2417)^{333}$ is divided by 9 is less than	2.	5
R.	The digit in the unit place of the number $183! + 3^{183}$ is greater than than	3.	6
S.	If sum of the coefficients of the first, second and third terms of the expansion of $\left(x^2 + \frac{1}{x}\right)^m$ is 46, then the index of the term that does not contain x is greater than	4.	7

	P	Q	R	S
(a)	1,3,4	1,2	1,2,3	3,4
(b)	2,3,4	1,2,3	2,3,4	1,3
(c)	1,2	3,4	1,4	2,3
(d)	2,3,4	1,2,3,4	1,2,3	1,2

Numerical Answer Type

17. The number of terms which are free from radical signs in the expansion of $(y^{1/5} + x^{1/10})^{55}$ is

18. If $(2r + 3)^{th}$ and $(r - 1)^{th}$ terms in the expansion of $(1 + x)^{15}$ have equal coefficients then r is

19. If the coefficient of the middle term in the binomial expansions of $(1 + \alpha x)^4$ and $(1 - \alpha x)^6$ is same, then -10α equals

20. The sum of the last 4 digits of the natural number 3^{100} is

❖❖

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No. of questions correct ……
Marks scored in percentage ……

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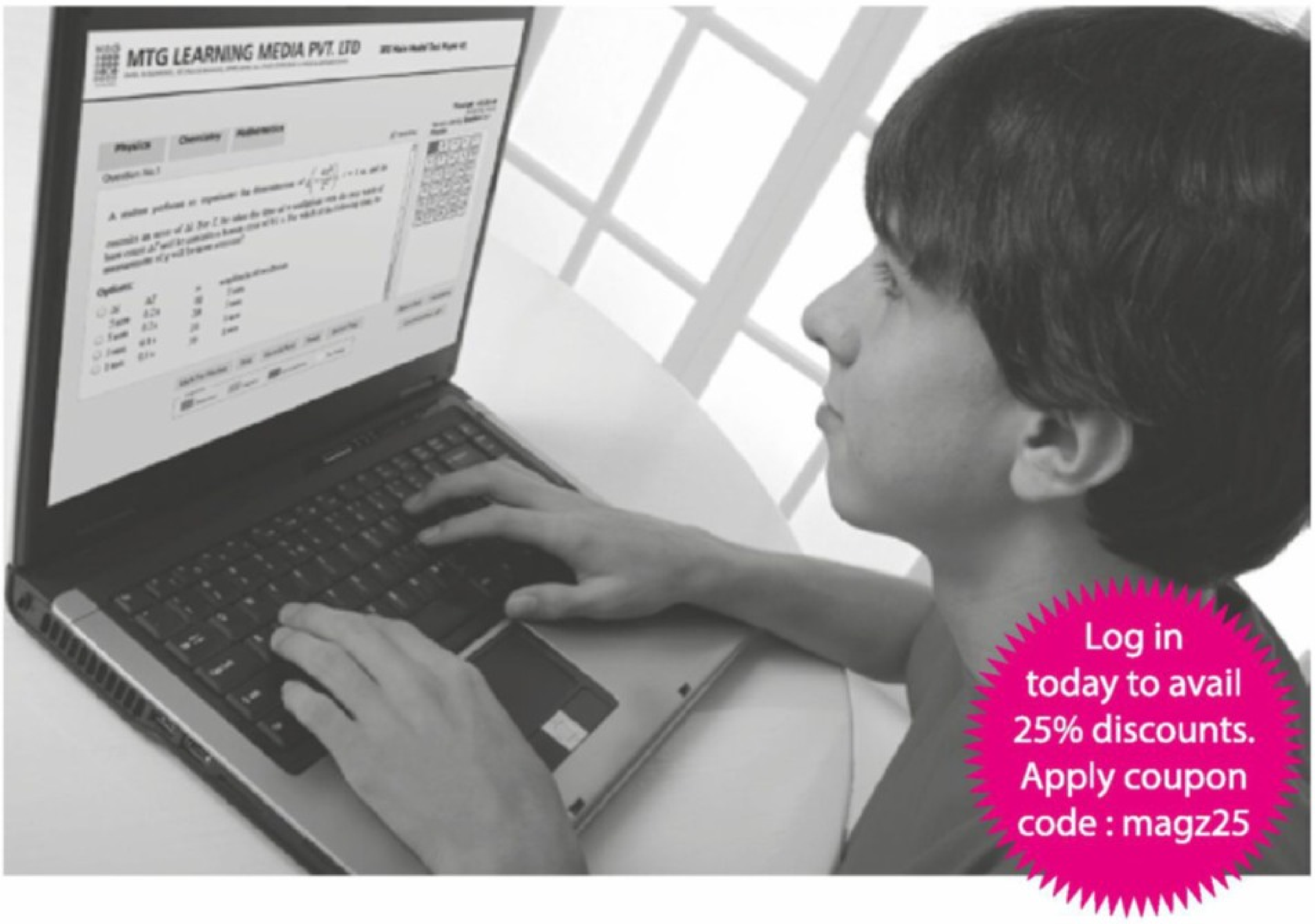

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CLASS-XII

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Series 6

Differential Equations

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) This question paper contains **36** questions.
(iii) Question **1-20** in Section-A are very short-answer-objective type questions carrying **1** mark each.
(iv) Question **21-26** in Section-B are short-answer type questions carrying **2** marks each.
(v) Question **27-32** in Section-C are long-answer-**I** type questions carrying **4** marks each.
(vi) Question **33-36** in Section-D are long-answer-**II** type questions carrying **6** marks each.

SECTION - A

1. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2=x\sin\left(\frac{dy}{dx}\right)$ is

(a) 1 (b) 2
(c) 3 (d) not defined

2. The differential equation whose solution is given by $c + 4\,yx = 0$ is

(a) $xy + y_1 = 0$ (b) $xy_1 + y = 0$
(c) $y_1 = 4xy$ (d) $xy_1 + 1 = 0$

3. The order of the differential equation whose solution is $y = a\cos x + b\sin x + ce^{-x}$ is

(a) 3 (b) 2
(c) 1 (d) none of these

4. The degree of $2\frac{d^3y}{dx^3}-5\left(\frac{dy}{dx}\right)^2+7y=x^2$ is

(a) undefined (b) 1
(c) 2 (d) 3

5. The differential equation which represent the family of curves $y = ae^{bx}$, where a and b are arbitrary constants, is

(a) $y' = y^2$ (b) $y'' = y\,y'$
(c) $y\,y'' = y'$ (d) $y\,y'' = (y')^2$

6. The differential equation satisfied by $y = \frac{A}{x} + B$ is (A, B are parameters)

(a) $x^2y_1 = y$ (b) $xy_1 + 2y_2 = 0$
(c) $xy_2 + 2y_1 = 0$ (d) none of these

7. The differential equation having solution as $y = 17e^x + ae^{-x}$ is

(a) $y'' - x = 0$ (b) $y'' - y = 0$
(c) $y' - y = 0$ (d) $y' - x = 0$

8. The general solution of $\frac{dy}{dx} = 2x\,e^{x^2-y}$ is

(a) $e^{x^2-y} = c$ (b) $e^{-y} + e^{x^2} = c$
(c) $e^y = e^{x^2} + c$ (d) $e^{x^2+y} = c$

9. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

(a) x/e^x (b) e^x/x (c) xe^x (d) e^x

10. The solution of the differential equation

$\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is

(a) $e^x = \frac{y^3}{3} + e^y + c$ (b) $e^y = \frac{x^2}{3} + e^x + c$

(c) $e^y = \frac{x^3}{3} + e^x + c$ (d) none of these

11. Find the differential equation corresponding to $y = Ae^x + Be^{-x}$.

12. Find the order and degree of the differential equation $y = px + \sqrt{a^2p^2 + b^2}$, where $p = \frac{dy}{dx}$.

13. Solve : $\frac{dy}{dx} = (e^x + 1)y$

14. Find the integrating factor of the differential equation $\left\{\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right\}\frac{dx}{dy} = 1 (x \neq 0)$.

15. Find the I.F. of $(1 - x^2)\frac{dy}{dx} + xy = ax$.

16. Find the differential equation of the family of all straight lines.

17. Find the solution of the differential equation $\frac{dy}{dx} = x^3 e^{-2y}$.

18. Determine the order of the differential equation $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$.

19. Find the differential equation of all non-vertical lines in a plane.

20. Solve the differential equation $\frac{dy}{dx} = 1 - x + y - xy$.

SECTION - B

21. Show that the function $y = Ax + \frac{B}{x}$ is a solution of the differential equation, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$.

22. Obtain the differential equation of the family of curves represented by $y = Ae^x + Be^{-x} + x^2$, where A and B are arbitrary constants.

OR

The slope of the tangent at a point $P(x, y)$ on a curve is $(-x/y)$. If the curve passes through the point $(3, -4)$, find the equation of the curve.

23. Solve : $(1 + xy)ydx + (1 - xy)xdy = 0$

24. Find the particular solution of $y' = y \cot 2x$, $y\left(\frac{\pi}{4}\right) = 2$.

OR

Verify that $y = ae^{3x} + be^{-x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$.

25. Solve : $\frac{dy}{dx} = -\frac{x + y\cos x}{1 + \sin x}$

26. Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = e^x;\ x > 0$.

SECTION - C

27. Solve : $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

28. Find the differential equation of the family of all circles touching the x-axis at the origin.

OR

Solve : $(x + y)^2\frac{dy}{dx} = a^2$

29. Solve the differential equation $(1 + e^{2x})dy + e^x(1 + y^2)dx = 0$. Given that $y = 1$, when $x = 0$.

30. Solve the following differential equation

$\frac{dy}{dx} + \sec x \cdot y = \tan x \left(0 \leq x < \frac{\pi}{2}\right)$.

OR

Form the differential equation corresponding to $(x - a)^2 + (y - b)^2 = r^2$ by eliminating a and b.

31. Show that the curve for which the normal at every point passes through a fixed point is a circle.

32. Find the particular solution of the differential equation $e^x\sqrt{1 - y^2}\,dx + \frac{y}{x}dy = 0$ given that $y = 1$ when $x = 0$.

Monthly Test Drive-5 CLASS XI ANSWER KEY

1.	(c)	2.	(c)	3.	(b)	4.	(b)
5.	(c)	6.	(d)	7.	(a, b)	8.	(a, b, c)
9.	(a, b, c)	10.	(b, c)	11.	(b, c)	12.	(a, b, c)
13.	(a, b, c)	14.	(d)	15.	(b)	16.	(d)
17.	(6)	18.	(5)	19.	(3)	20.	(3)

SECTION - D

33. If the tangent at any point P of a curve meets the axis of X in T. Find the curve for which $OP = PT$, O being the origin.

34. Solve the following differential equation

$$\frac{dy}{dx} + y\cot x = x^2 \cot x + 2x$$

Also find the particular solution, given that $y = 0$ when $x = \frac{\pi}{2}$.

OR

Solve the following differential equation :

$$\left(\sqrt{1+x^2+y^2+x^2y^2}\right)dx + xy\,dy = 0.$$

35. Find the particular solution of $(x + y)dy + (x - y)dx = 0$, given that $y = 1$ when $x = 1$.

OR

Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.

36. Solve $(x^2 + xy)\,dy = (x^2 + y^2)\,dx$. Find the particular solution given that $y = 0$ when $x = 1$.

SOLUTIONS

1. (d) : Since, the term $\sin\left(\frac{dy}{dx}\right)$ occurs in the given differential equation, therefore, its degree is not defined.

2. (b) : We have $c + 4yx = 0 \Rightarrow xy = -c/4$ ...(i)

Differentiating (i) w.r.t. x, we get $xy_1 + y = 0$

3. (a) : $y = a\cos x + b\sin x + ce^{-x}$ is a three parameter family of curve so its differential equation is third order differential equation.

4. (b) : Highest order derivative is $\frac{d^3y}{dx^3}$. So, its degree is 1.

5. (d) : We have $y = ae^{bx} \Rightarrow \log y = \log a + bx$ ...(i)

Differentiating (i) w.r.t. x, we get $\frac{1}{y}y' = b$ ...(ii)

Again differentiating (ii) w.r.t. x, we get

$$\frac{y''}{y} - \frac{1}{y^2}(y')^2 = 0 \Rightarrow y\,y'' = (y')^2$$

6. (c) : Given relation is $y = \frac{A}{x} + B$

Differentiating w.r.t. x, we get

$y_1 = -\frac{A}{x^2}$ or $x^2 y_1 = -A$

Again differentiating w.r.t. x, we get

$x^2y_2 + y_1\,2x = 0$ or $xy_2 + 2y_1 = 0$

7. (b) : $y' = 17e^x - ae^{-x}$

and $y'' = 17e^x + ae^{-x} = y \Rightarrow y'' - y = 0$

8. (c) : $\frac{dy}{dx} = 2xe^{x^2-y} \Rightarrow e^y dy = 2xe^{x^2}\,dx$

On integrating, we get $e^y = e^{x^2} + c$

9. (b) : $\frac{dy}{dx} + y = \frac{1+y}{x}$

$$\Rightarrow \frac{dy}{dx} + y = \frac{1}{x} + \frac{y}{x} \Rightarrow \frac{dy}{dx} + y - \frac{y}{x} = \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} + \left(1 - \frac{1}{x}\right)y = \frac{1}{x}$$

$$\text{I.F.} = e^{\int\left(1-\frac{1}{x}\right)dx} = e^{x-\log x} = e^x\,e^{-\log x} = e^x\,e^{\log x^{-1}}$$

$$= e^x\,x^{-1} = e^x/x$$

10. (c) : Given differential equation is

$$\frac{dy}{dx} = \frac{e^x + x^2}{e^y} \Rightarrow e^y dy = (e^x + x^2)\,dx$$

Integrating, we get $e^y = e^x + \frac{x^3}{3} + c$

11. We have, $y = Ae^x + Be^{-x}$ ...(i)

Differentiating (i) w.r.t. x, we get $\frac{dy}{dx} = Ae^x - Be^{-x}$

Differentiating again w.r.t. x, we get

$\frac{d^2y}{dx^2} = Ae^x + Be^{-x} = y$, which is the required differential equation.

12. We have, $y = px + \sqrt{a^2p^2 + b^2}$

$\Rightarrow y - px = \sqrt{a^2p^2 + b^2}$

$\Rightarrow (y - px)^2 = a^2p^2 + b^2$ [On squaring both sides]

$\Rightarrow y^2 + x^2p^2 - 2xyp = a^2p^2 + b^2$

$\Rightarrow (x^2 - a^2)p^2 - 2xyp + (y^2 - b^2) = 0$

$$\Rightarrow (x^2 - a^2)\left(\frac{dy}{dx}\right)^2 - 2xy\cdot\left(\frac{dy}{dx}\right) + (y^2 - b^2) = 0$$

Clearly, it is a differential equation of order 1 and degree 2.

13. We have, $\frac{dy}{dx} = (e^x + 1)y \Rightarrow \frac{dy}{y} = (e^x + 1)\,dx$

On integrating both sides, we get $\int\frac{dy}{y} = \int(e^x + 1)\,dx$

$\Rightarrow \log y = e^x + x + c$

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14. We have $\frac{dy}{dx}+\frac{1}{\sqrt{x}}\cdot y=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

This is of the form $\frac{dy}{dx}+Py=Q$, where $P=\frac{1}{\sqrt{x}}$ and $Q=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$.

$\therefore$ I.F. $=e^{\int Pdx}=e^{\int\frac{1}{\sqrt{x}}dx}=e^{2\sqrt{x}}$

15. We have, $(1-x^2)\frac{dy}{dx}+xy=ax$

$\Rightarrow \frac{dy}{dx}+\frac{x}{1-x^2}y=\frac{ax}{1-x^2}$

$\therefore$ I.F. $=e^{\int\frac{x}{1-x^2}dx}=e^{-\frac{1}{2}\int\frac{-2x}{1-x^2}dx}$

$=e^{-\frac{1}{2}\log(1-x^2)}=e^{\log(1-x^2)^{-1/2}}=(1-x^2)^{-1/2}=\frac{1}{\sqrt{1-x^2}}$

16. The general equation of the family of all straight lines is given by $y = mx + c$, where m and c are parameters.

Now, $y=mx+c \Rightarrow \frac{dy}{dx}=m \Rightarrow \frac{d^2y}{dx^2}=0$

So, the required differential equation is $\frac{d^2y}{dx^2}=0$

17. We have, $\frac{dy}{dx}=x^3e^{-2y} \Rightarrow e^{2y}\,dy=x^3dx$

On integrating, we get $\frac{e^{2y}}{2}=\frac{x^4}{4}+c'$

$\Rightarrow 2e^{2y}=x^4+c$, where $c=4c'$

18. We have $\frac{d^2y}{dx^2}=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

On squaring, we get $\left(\frac{d^2y}{dx^2}\right)^2=1+\left(\frac{dy}{dx}\right)^2$

Clearly, order of differential equation is 2.

19. The equation of the family of non-vertical lines in a plane is $ax + by = 1$, where $b \neq 0$ and $a \in R$.

Differentiating w.r.t. x, we get

$$a+b\frac{dy}{dx}=0$$

Differentiating again w.r.t. x, we get

$$b\frac{d^2y}{dx^2}=0$$

$\Rightarrow \frac{d^2y}{dx^2}=0$ is the required differential equation.

20. We have $\frac{dy}{dx}=(1-x)(1+y) \Rightarrow \frac{dy}{1+y}=(1-x)dx$

$\Rightarrow \int\frac{dy}{1+y}=\int(1-x)dx \Rightarrow \log|1+y|=x-\frac{x^2}{2}+c$

21. The given function is $y=Ax+\frac{B}{x}$

$\Rightarrow yx=Ax^2+B$ ...(i)

Differentiating (i) w.r.t. x, we get

$\frac{xdy}{dx}+y=2Ax \Rightarrow \frac{dy}{dx}+\frac{y}{x}=2A$ ...(ii)

Differentiating (ii) w.r.t. x, we get

$$\frac{d^2y}{dx^2}+\frac{x\cdot\frac{dy}{dx}-y\cdot 1}{x^2}=0 \Rightarrow x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$$

Hence, $y=Ax+\frac{B}{x}$ is a solution of above differential equation.

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

4			7										2				9			
6		7	9			1	4	3					3			5	8	2	6	1
2						7	5								2				4	
			2					1					7					3	1	5
5	9		8		1		2						5		6	1			2	4
		2			9		3					9			3		5		8	
						2				5	1	3							9	
1	2				3			4	9	3		5	1	6						
				4			1	5	6		7				7		1		3	
							9													
							2	7		4		9	5							
						5		8				6	7	3						
					8				8			1			3	8	2			4
8					2	7	3			9										1
	7	3		4			8	6	5		4				5	4			9	6
5	2				7	8							7	1			6			9
7				3			9										7			8
9	3	6	5									4			9		5		6	
4					3							2		4					3	
		8	4	6	1		7					9	8	6						
	6							5							6					

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

22. We have $y = Ae^x + Be^{-x} + x^2$ ...(i)

Differentiating (i) w.r.t. x, we get

$$\frac{dy}{dx} = Ae^x - Be^{-x} + 2x \quad \text{...(ii)}$$

Differentiating (ii) w.r.t. x, we get

$$\frac{d^2y}{dx^2} = Ae^x + Be^{-x} + 2 \quad \text{...(iii)}$$

On subtracting (iii) from (i), we get

$y - \frac{d^2y}{dx^2} = x^2 - 2 \Rightarrow \frac{d^2y}{dx^2} - y + x^2 - 2 = 0$, which is the required differential equation.

OR

Given, $\frac{dy}{dx} = -\frac{x}{y} \Rightarrow y\,dy + x\,dx = 0$

Integrating both sides, we get

$\frac{y^2}{2} + \frac{x^2}{2} = c' \Rightarrow y^2 + x^2 = 2c'$

$\Rightarrow x^2 + y^2 = c$, where $c = 2c'$

Since it passes through $(3, -4)$.

$\therefore (3)^2 + (-4)^2 = c \Rightarrow c = 25$

Hence, the required equation of the curve is $x^2 + y^2 = 25$.

23. Given, $(1 + xy)y dx + (1 - xy)x dy = 0$

$\Rightarrow (ydx + xdy) + xy(ydx - xdy) = 0$

$\Rightarrow d(xy) = xy(xdy - ydx)$

$\Rightarrow \frac{d(xy)}{x^2y^2} = \frac{xdy - ydx}{xy} \Rightarrow \frac{d(xy)}{(xy)^2} = \frac{dy}{y} - \frac{dx}{x}$

Integrating both sides, we get $-\frac{1}{xy} = \log y - \log x + c$

$\Rightarrow -\frac{1}{xy} = \log\left(\frac{y}{x}\right) + c$, which is the required solution.

24. We have, $y' = y \cot 2x \Rightarrow \frac{dy}{dx} = y \cot 2x$

$\Rightarrow \frac{dy}{y} = \cot 2x\,dx$

Integrating both sides, we get $\int \frac{dy}{y} = \int \cot 2x\,dx$

$\Rightarrow \log|y| = \frac{1}{2}\log|\sin 2x| + \log c$

$\Rightarrow \log|y| = \log\left|\sqrt{\sin 2x}\right| + \log c$

$\Rightarrow \log|y| = \log\left|c\sqrt{\sin 2x}\right| \Rightarrow y = c\sqrt{\sin 2x}$

Given $y\left(\frac{\pi}{4}\right) = 2$

$\Rightarrow 2 = c\sqrt{\sin\frac{\pi}{2}} \Rightarrow c = 2 \quad \therefore \quad y = 2\sqrt{\sin 2x}$

OR

We have $y = ae^{3x} + be^{-x}$

$\therefore \frac{dy}{dx} = 3ae^{3x} - be^{-x}$ and $\frac{d^2y}{dx^2} = 9ae^{3x} + be^{-x}$

Now, $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = (9ae^{3x} + be^{-x})$

$-2(3ae^{3x} - be^{-x}) - 3(ae^{3x} + be^{-x}) = 0$

Hence, $y = ae^{3x} + be^{-x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$.

25. We have, $\frac{dy}{dx} = -\frac{x + y\cos x}{1 + \sin x}$

$\Rightarrow \frac{dy}{dx} + \frac{\cos x}{1 + \sin x}y = -\frac{x}{1 + \sin x}$ ...(i)

$\therefore$ I.F. $= e^{\int \frac{\cos x}{1 + \sin x}dx} = e^{\log(1 + \sin x)} = (1 + \sin x)$

Hence, the solution is

$y(1 + \sin x) = \int\left(\frac{-x}{1 + \sin x}\right)(1 + \sin x)\,dx + c$

$\Rightarrow y(1 + \sin x) = -\frac{x^2}{2} + c \Rightarrow y = \frac{2c - x^2}{2(1 + \sin x)}$

26. We have $\frac{dy}{dx} + \frac{1}{x}y = e^x; (x > 0)$

This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$

where, $P = \frac{1}{x}$ and $Q = e^x$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$

$\therefore$ The solution is given by

$yx = \int xe^x dx + c$ or $yx = e^x(x - 1) + c$

This is the required solution of the given differential equation.

27. We have, $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

$\Rightarrow \frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \Rightarrow \frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$ ...(i)

This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{\sqrt{x}}$ and $Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$.

$\therefore$ I.F. $= e^{\int P\,dx} = e^{\int \frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$

Hence, the required solution is

$$ye^{2\sqrt{x}} = \int e^{2\sqrt{x}}\cdot\frac{e^{-2\sqrt{x}}}{\sqrt{x}}dx+c$$

$$\Rightarrow\ ye^{2\sqrt{x}} = \int\frac{1}{\sqrt{x}}dx+c \Rightarrow ye^{2\sqrt{x}} = 2\sqrt{x}+c$$

$$\Rightarrow\ y=(2\sqrt{x}+c)e^{-2\sqrt{x}}$$

28. Equation of a circle with centre $(0, a)$ and radius a is $x^2+(y-a)^2=a^2$ or $x^2+y^2=2ay$, ...(i)
where a is a parameter.

Differentiating both sides of (i) w.r.t. x, we get

$$2x+2y\frac{dy}{dx}=2a\frac{dy}{dx} \Rightarrow a\frac{dy}{dx}=x+y\frac{dy}{dx}$$

$$\Rightarrow\ a=\frac{x}{\left(\frac{dy}{dx}\right)}+y \qquad \text{...(ii)}$$

Putting the value of a from (ii) in (i), we get

$$x^2+y^2=2y\left\{\frac{x}{\left(\frac{dy}{dx}\right)}+y\right\} \Rightarrow (x^2-y^2)=\frac{2xy}{\left(\frac{dy}{dx}\right)}$$

$\Rightarrow\ (x^2-y^2)\frac{dy}{dx}-2xy=0,$ which is the required differential equation.

OR

The given equation is $(x+y)^2\frac{dy}{dx}=a^2$ ...(i)

Let $x+y=v$. Then,

$$1+\frac{dy}{dx}=\frac{dv}{dx}\Rightarrow\frac{dy}{dx}=\frac{dv}{dx}-1 \qquad \text{...(ii)}$$

Using (ii) in (i), we get

$$v^2\left(\frac{dv}{dx}-1\right)=a^2 \Rightarrow v^2\frac{dv}{dx}=a^2+v^2$$

$\Rightarrow\ v^2\,dv=(a^2+v^2)dx$

$\Rightarrow\ \frac{v^2}{v^2+a^2}dv=dx$ [By separating the variables]

$$\Rightarrow\ \left(1-\frac{a^2}{v^2+a^2}\right)dv=dx$$

Integrating both sides, we get

$$\int 1\cdot dv-a^2\int\frac{1}{v^2+a^2}dv=\int dx+c$$

$$\Rightarrow\ v-a\tan^{-1}\left(\frac{v}{a}\right)=x+c$$

Hence, $(x+y)-a\tan^{-1}\left(\frac{x+y}{a}\right)=x+c$, is the required solution.

29. We have $(1+e^{2x})dy+e^x(1+y^2)dx=0$

$$\Rightarrow\ \frac{dy}{1+y^2}+\frac{e^x}{1+e^{2x}}dx=0$$

Integrating, we get

$$\tan^{-1}y+\int\frac{dt}{1+t^2}=c \qquad (\text{Where } t=e^x)$$

$\Rightarrow\ \tan^{-1}y+\tan^{-1}t=c \Rightarrow \tan^{-1}y+\tan^{-1}(e^x)=c$

Now, when $x=0$, $y=1$ [Given]

$\therefore\ \tan^{-1}1+\tan^{-1}(e^0)=c$

$$\text{or}\quad c=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2} \quad\therefore\quad \tan^{-1}y+\tan^{-1}e^x=\frac{\pi}{2}$$

This is the required solution of the given differential equation.

30. We have $\frac{dy}{dx}+\sec x\cdot y=\tan x$ ...(i)

This is a linear differential equation of the form $\frac{dy}{dx}+Py=Q$, where $P=\sec x$ and $Q=\tan x$

Now, $\text{I.F.}=e^{\int Pdx}=e^{\int\sec x dx}$

$$=e^{\log(\sec x+\tan x)}=\sec x+\tan x$$

$\therefore$ The solution of (i) is given by

$$y\cdot(\sec x+\tan x)=\int\tan x(\sec x+\tan x)dx+c$$

$$=\int(\sec x\tan x+\tan^2x)dx+c$$

$$=\int\sec x\tan x\,dx+\int\sec^2x\,dx-\int 1\cdot dx+c$$

$\therefore\ y(\sec x+\tan x)=\sec x+\tan x-x+c$, which is the required solution of the given differential equation.

OR

We have, $(x-a)^2+(y-b)^2=r^2$ ...(i)

Differentiating (i) w.r.t. x, we have

$$2(x-a)+2(y-b)\frac{dy}{dx}=0$$

$$\Rightarrow\ (x-a)+(y-b)\frac{dy}{dx}=0 \qquad \text{...(ii)}$$

Differentiating (ii) w.r.t. x, we have

$$1+(y-b)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0 \qquad \text{...(iii)}$$

From (ii) and (iii), we get

$$y-b=-\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} \text{ and } x-a=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]\frac{dy}{dx}}{\frac{d^2y}{dx^2}}$$

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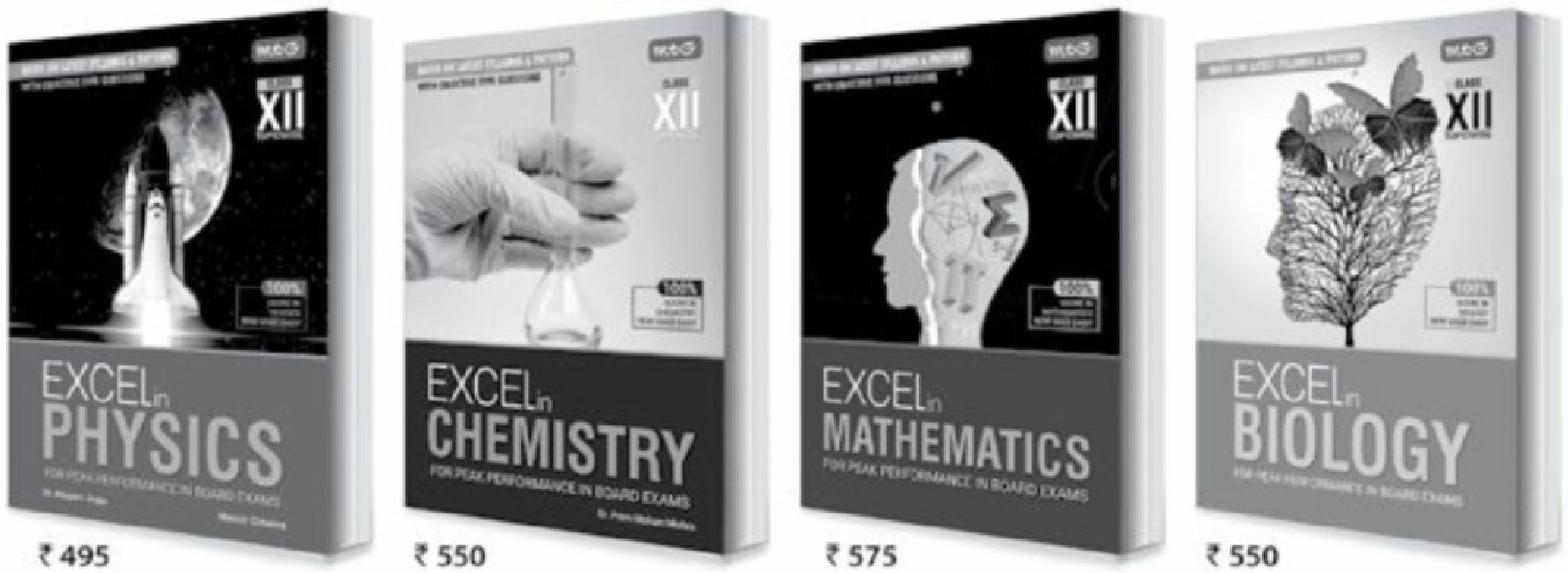

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Now, substituting these in the given equation, we obtain

$$\left\{\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]\frac{dy}{dx}}{\frac{d^2y}{dx^2}}\right]^2+\left\{\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^2=r^2$$

$\Rightarrow \quad \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3=r^2\left(\frac{d^2y}{dx^2}\right)^2$, which is the required differential equation.

31. Let $P(x, y)$ be an arbitrary point on the given curve. The equation of the normal to the given curve at (x, y) is $Y-y=-\frac{1}{\frac{dy}{dx}}(X-x)$

It is given that the normal at every point passes through a fixed point (α, β) (say).

Therefore, $\beta-y=-\frac{dx}{dy}(\alpha-x)$

$\Rightarrow \quad (x-\alpha)dx+(y-\beta)dy=0$

Integrating both sides, we get

$$\int(x-\alpha)dx+\int(y-\beta)dy=c$$

$$\Rightarrow \quad \frac{(x-\alpha)^2}{2}+\frac{(y-\beta)^2}{2}=c$$

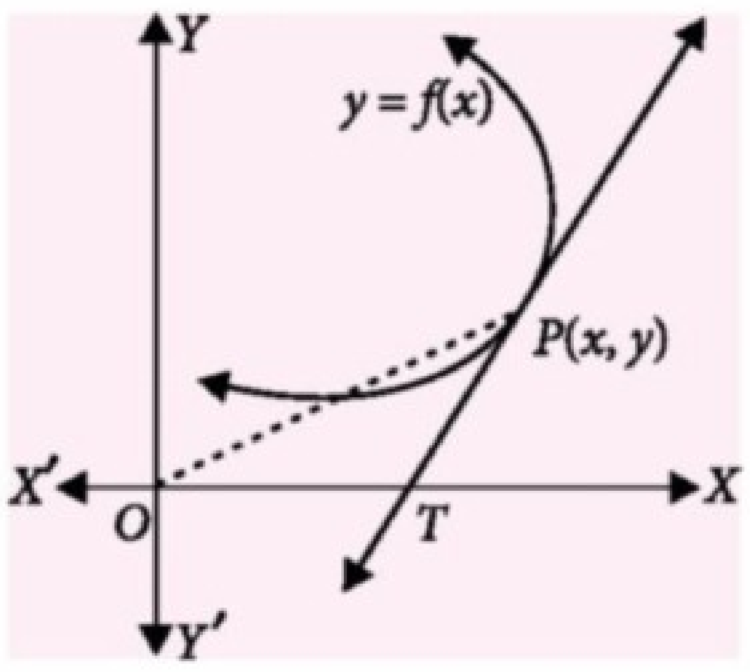
$\Rightarrow \quad (x-\alpha)^2+(y-\beta)^2=r^2$, where $r^2=2c$

Clearly, this equation represents a circle, having centre at (α, β) and radius r.

32. *Refer to answer 43, Page no. 228 of MTG CBSE Champion Mathematics, Class-12*

33. Let $y=f(x)$ be given curve and let $P(x, y)$ be a point on it. The equation of the tangent at P is

$$Y-y=\frac{dy}{dx}(X-x) \qquad \text{...(i)}$$



This meets X-axis at T. So, the x-coordinate of T is obtained by putting $Y=0$ in (i).

Putting $Y=0$ in (i), we get

$$0-y=\frac{dy}{dx}(X-x)$$

$$\Rightarrow X=x-y\frac{dx}{dy}$$

Thus, the coordinates of T are $\left(x-y\frac{dx}{dy}, 0\right)$

$$\therefore \quad PT=\sqrt{\left\{x-\left(x-y\frac{dx}{dy}\right)\right\}^2+(y-0)^2}$$

$$=\sqrt{y^2\left(\frac{dx}{dy}\right)^2+y^2}$$

Now, $PT=OP$ [Given]

$$\Rightarrow \quad \sqrt{y^2+y^2\left(\frac{dx}{dy}\right)^2}=\sqrt{x^2+y^2}$$

$$\Rightarrow \quad y^2+y^2\left(\frac{dx}{dy}\right)^2=x^2+y^2 \Rightarrow y^2\left(\frac{dx}{dy}\right)^2=x^2$$

$$\Rightarrow \quad y\frac{dx}{dy}=\pm x \quad \Rightarrow \quad \frac{dx}{x}=\pm\frac{dy}{y}$$

$\Rightarrow \quad \log x=\pm \log y+\log c$

$\Rightarrow \quad \log x=\log(cy)$ or $\log x=\log\left(\frac{c}{y}\right)$

$\Rightarrow \quad x=cy$ or $x=\frac{c}{y}$, are the equations of curve.

34. We have $\frac{dy}{dx}+y\cot x=x^2\cot x+2x$ ...(i)

This is a linear differential equation of the form $\frac{dy}{dx}+Py=Q$, where $P=\cot x$ and $Q=x^2\cot x+2x$

$\therefore \quad \text{I.F.}=e^{\int P dx}=e^{\int \cot x\, dx}=e^{\log \sin x}=\sin x$

$\therefore \quad$ The solution of (i) is given by

$$y\cdot\sin x=\int(x^2\cot x+2x)\sin x dx+c$$

$$=\int x^2\cos x dx+2\int x\sin x dx+c$$

$$=x^2\sin x-\int 2x\cdot\sin x dx+2\int x\sin x dx+c$$

$$=x^2\sin x+c$$

$\Rightarrow \quad y=x^2+c\cdot\operatorname{cosec} x$ ...(ii)

Since, $y=0$, when $x=\frac{\pi}{2}$

$\therefore \quad$ From (ii), $0=\frac{\pi^2}{4}+c \Rightarrow c=-\frac{\pi^2}{4}$

$\therefore \quad y=x^2-\frac{\pi^2}{4}\operatorname{cosec} x$, which is the required particular solution of the given differential equation.

OR

Refer to answer 90, Page no. 238 of MTG CBSE Champion Mathematics, Class-12

35. We have, $(x+y)dy+(x-y)dx=0$

$\Rightarrow (x+y)dy=-(x-y)dx \Rightarrow \frac{dy}{dx}=\frac{y-x}{x+y}$ ...(i)

This is a homogeneous linear differential equation.

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ ...(ii)

From (i) and (ii), we get

$$v + x\frac{dv}{dx} = \frac{(v-1)x}{(v+1)x} \Rightarrow x\frac{dv}{dx} = \frac{v-1}{v+1} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v-1-v^2-v}{v+1} \Rightarrow \frac{v+1}{1+v^2}dv = -\frac{dx}{x}$$

$$\Rightarrow \int\frac{1}{1+v^2}dv + \int\frac{v}{1+v^2}dv = -\int\frac{dx}{x}$$

$$\Rightarrow \tan^{-1}v + \frac{1}{2}\int\frac{2v}{1+v^2}dv = -\log|x| + c$$

$$\Rightarrow \tan^{-1}v + \frac{1}{2}\log|1+v^2| = -\log|x| + c$$

$$\Rightarrow \tan^{-1}\frac{y}{x} + \frac{1}{2}\log\left|1+\frac{y^2}{x^2}\right| = -\log|x| + c \quad \text{...(iii)}$$

Putting $y = 1$ and $x = 1$ in equation (iii), we get

$$\tan^{-1}1 + \frac{1}{2}\log\left|1+\frac{1}{1}\right| = -\log|1| + c$$

$$\Rightarrow \quad c = \frac{\pi}{4} + \frac{1}{2}\log 2$$

Putting $c = \frac{\pi}{4} + \log\sqrt{2}$ in equation (iii), we get

$$\tan^{-1}\frac{y}{x} + \frac{1}{2}\log\left[1+\frac{y^2}{x^2}\right] + \log|x| = \frac{\pi}{4} + \log\sqrt{2}$$

OR

Refer to answer 17, Page no. 222 of MTG CBSE Champion Mathematics, Class-12

36. We have, $(x^2 + xy)\,dy = (x^2 + y^2)\,dx$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy} \quad \text{... (i)}$$

This is a homogeneous linear differential equation

Put $y = vx \Rightarrow \frac{dy}{dx} = v\cdot 1 + x\frac{dv}{dx}$

$\therefore$ (i) becomes

$$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{x^2+x\cdot vx} = \frac{1+v^2}{1+v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+v^2}{1+v} - v = \frac{1-v}{1+v} \quad \Rightarrow \quad \frac{1+v}{1-v}dv = \frac{dx}{x}$$

Integrating both sides, we get

$$\int\left[-1+\frac{2}{1-v}\right]dv = \log x + c$$

$$\Rightarrow \; -v - 2\log(1-v) = \log x + c$$

$$\Rightarrow \quad \frac{-y}{x} - 2\log\left(1-\frac{y}{x}\right) = \log x + c \quad \text{...(ii)}$$

Putting $x = 1$, $y = 0$ in (ii), we get $c = 0$

$$\Rightarrow \quad \frac{-y}{x} - 2\left[\log(x-y) - \log x\right] = \log x$$

$\Rightarrow \quad \frac{-y}{x} - 2\log(x-y) + \log x = 0,$ which is the required particular solution.

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MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 5 : Application of Derivatives

Time Taken : 60 Min.

Only One Option Correct Type

1. The equation of the tangent to the curve $y = e^{-|x|}$ at the point where the curve cuts the line $x = 1$ is

(a) $x + y = e$ (b) $e(x + y) = 1$

(c) $y + ex = 1$ (d) none of these

2. If the function $f(x) = \cos|x| - 2ax + b$ increases along the entire number scale, the range of values of a is given by

(a) $a \le b$ (b) $a = b/2$

(c) $a \le -1/2$ (d) $a \ge 3/2$

3. Let $f(x) = \tan x - \tan 2x$. Then

(a) f has no critical point

(b) f has minimum at $x = \cos^{-1}(\sqrt{10}-1)/4$

(c) f has minimum at $x = \cos^{-1}(\sqrt{10}-\sqrt{2})/4$

(d) f has maximum at $x = \cos^{-1}(\sqrt{10}-\sqrt{2})/4$

4. By LMVT, which of the following is true for $x > 1$?

(a) $1 + x\ln x < x < 1 + \ln x$

(b) $1 + \ln x < x < 1 + x\ln x$

(c) $x < 1 + x\ln x < 1 + \ln x$

(d) $1 + \ln x < 1 + x\ln x < x$

5. The diagonal of a square is changing at the rate of 0.5 cm/sec. Then the rate of change of area, when the area is 400 cm^2, is equal to

(a) $20\sqrt{2}$ cm^2/sec (b) $10\sqrt{2}$ cm^2/sec

(c) $\frac{1}{10\sqrt{2}}$ cm^2/sec (d) $\frac{10}{\sqrt{2}}$ cm^2/sec

6. If $f(x) = x^{3/2}(3x - 10)$, $x > 0$, then $f(x)$ is increasing in

(a) $(-\infty, -1) \cup (1, \infty)$ (b) $[2, \infty)$

(c) $(-\infty, -1) \cup [1, \infty)$ (d) $(-\infty, 0] \cup (2, \infty)$

One or More Than One Option(s) Correct Type

7. The normal to the curve $5x^5 - 10x^3 + x + 2y + 6 = 0$ at the point $P(0, -3)$ is tangent to the curve at the point(s)

(a) $(2, -44)$ (b) $(-2, 38)$

(c) $(1, -1)$ (d) $(-1, -5)$

8. A point on the ellipse $4x^2 + 9y^2 = 36$, where the tangent is equally inclined to the axes is

(a) $\left(\frac{9}{\sqrt{13}}, \frac{4}{\sqrt{13}}\right)$ (b) $\left(-\frac{9}{\sqrt{13}}, \frac{4}{\sqrt{13}}\right)$

(c) $\left(\frac{9}{\sqrt{13}}, -\frac{4}{\sqrt{13}}\right)$ (d) none of these

9. Let $f(x) = \begin{cases} x^3 + x^2 - 10x, & -1 \le x < 0 \\ \sin x, & 0 \le x < \pi/2 \\ 1 + \cos x, & \pi/2 \le x \le \pi \end{cases}$

then $f(x)$ has

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(a) local maxima at $x = \pi/2$
(b) local minima at $x = \pi/2$
(c) absolute maxima at $x = 0$
(d) absolute maxima at $x = \pi/2$

10. If $f(x)$ be a thrice differentiable function such that $f'(x) = x^2 - x + 1$ for $x = -1, 0, 1$ then
(a) $f''(x) - 2x + 1$ is not one-one in $(-1, 1)$.
(b) $f''(x) - 2 = 0$ has at most one root in $(0, 1)$.
(c) $f''(x) = 2$ for some $x \in (-1, 0)$.
(d) $f'''(x) - 2 = 0$ has at least one root in $(-1, 1)$.

11. If $\phi(x) = f(x) + f(2a - x)$ and $f''(x) > 0$, $a > 0$, $0 \le x \le 2a$, then
(a) $\phi(x)$ increases in $(a, 2a)$
(b) $\phi(x)$ increases in $(0, a)$
(c) $\phi(x)$ decreases in $(a, 2a)$
(d) $\phi(x)$ decreases in $(0, a)$

12. If $f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \le x \le 2 \\ 37 - x, & 2 < x \le 3 \end{cases}$, then
(a) $f(x)$ is increasing on $[-1, 2]$
(b) $f(x)$ is continuous on $[-1, 3]$
(c) $f'(2)$ does not exist
(d) $f(x)$ has the minimum value at $x = 2$

13. Let $f(x) = 2x^2 - \ln|x|, x \ne 0$, then $f(x)$ is monotonically
(a) increasing in $\left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$
(b) decreasing in $\left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$
(c) increasing in $\left(-\infty, \frac{1}{2}\right) \cup \left(0, \frac{1}{2}\right)$
(d) decreasing in $\left(-\infty, -\frac{1}{2}\right) \cup \left(0, \frac{1}{2}\right)$

Comprehension Type

Consider $f(x) = |x - 1| + |x - 3| + |5 - x|, \forall x \in R$.

14. If $f(x)$ increases, then $x \in$
(a) $(1, \infty)$ (b) $(3, \infty)$
(c) $(5, \infty)$ (d) $(1, 3)$

15. If $f(x)$ decreases, then $x \in$
(a) $(-\infty, 1)$ (b) $(-\infty, 3)$ (c) $(-\infty, 5)$ (d) $(3, 5)$

Matrix Match Type

16. Match the columns:

	Column-I	Column-II	
P.	$f(x) = \cos \pi x + 10x + 3x^2 + x^3$, $-2 \le x \le 3$. The absolute minimum value of $f(x)$ is	1.	3/4
Q.	If $x \in [-1, 1]$, then the minimum value of $f(x) = x^2 + x + 1$, is	2.	2
R.	Let $f(x) = (4/3)x^3 - 4x$, $0 \le x \le 2$. Then, the global minimum value of the function is	3.	–15
S.	Let $f(x) = 6 - 12x + 9x^2 - 2x^3$, $1 \le x \le 4$. Then the absolute maximum value of $f(x)$ in the interval is	4.	– 8/3

	P	Q	R	S
(a)	2	1	3	4
(b)	3	1	4	2
(c)	3	2	1	4
(d)	1	2	3	4

Numerical Answer Type

17. The maximum slope of the curve $-x^3 + 3x^2 + 2x - 27$ is

18. The area of the largest circle with centre $\left(\frac{1}{2}, 0\right)$ inscribed in the ellipse $x^2 + 2y^2 = 2$ is $\frac{m\pi}{n}$, where m and n are relatively prime positive integers. Then $m + n$ is

19. If the approximate value of $\log_{10}(4.04)$ is $0.abcdef$. It is given that $\log_{10} 4 = 0.6021$ and $\log_{10} e = 0.4343$, then the value of a must be

20. If $A > 0$, $B > 0$ and $A + B = \pi/3$ then the maximum value of $3 \tan A \tan B$ is

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No. of questions correct ......
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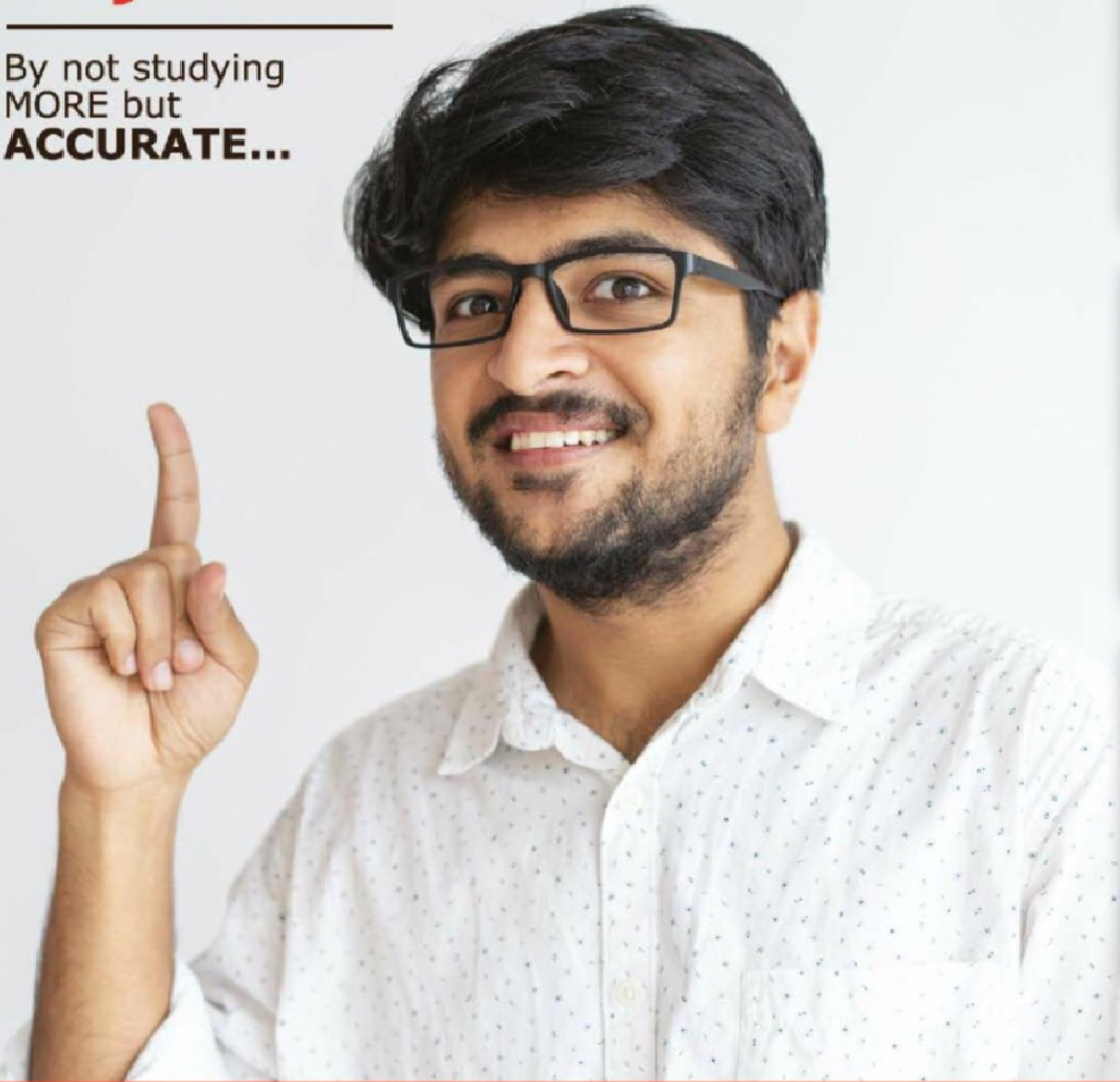

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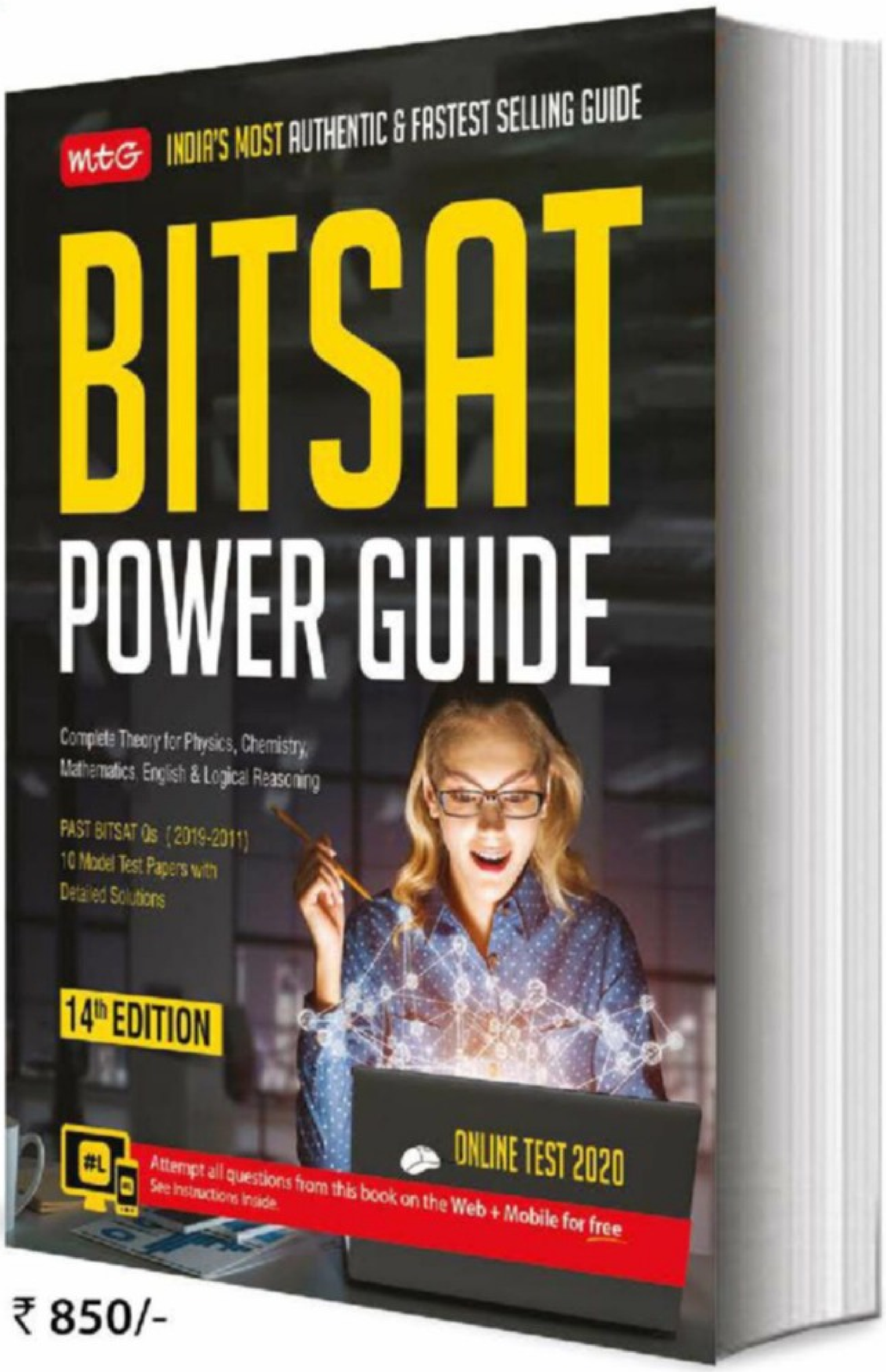
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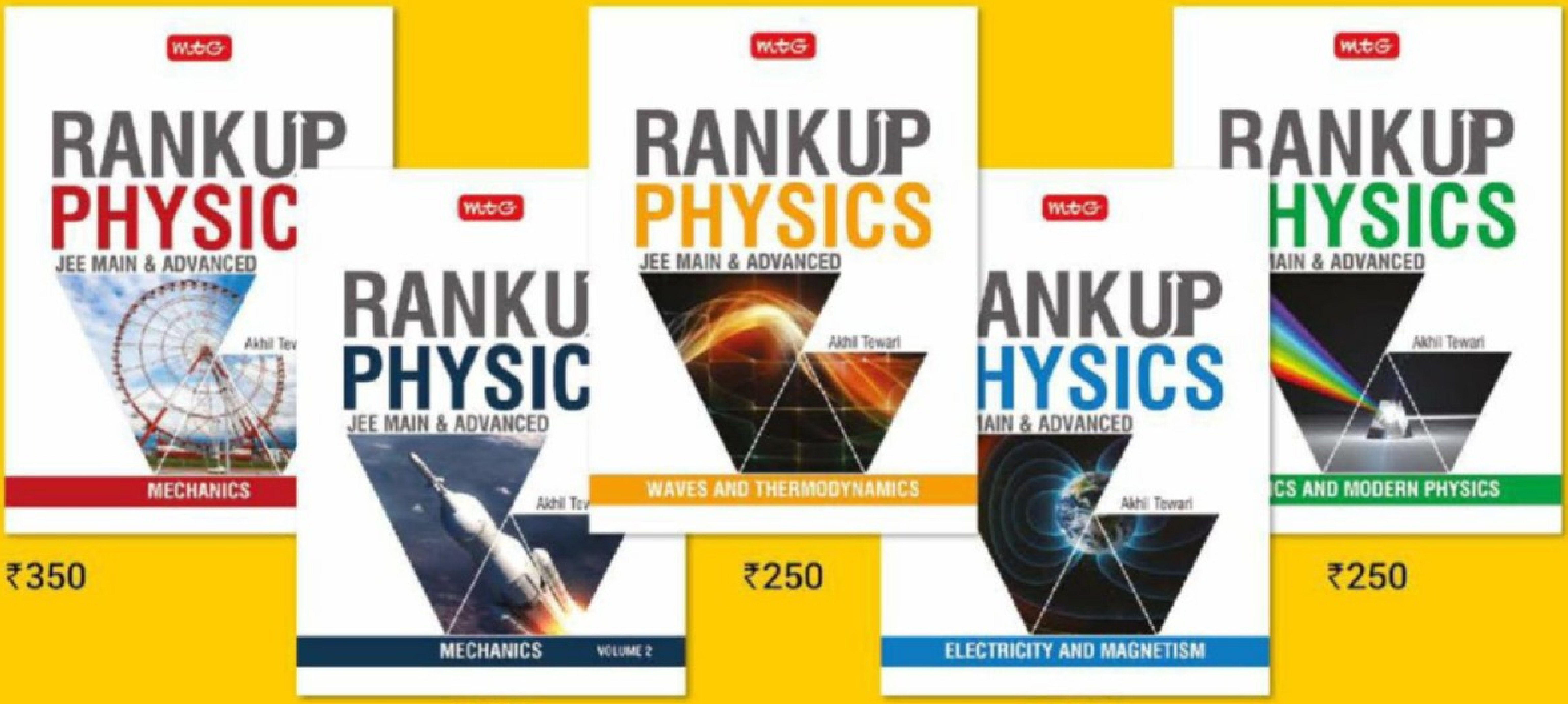
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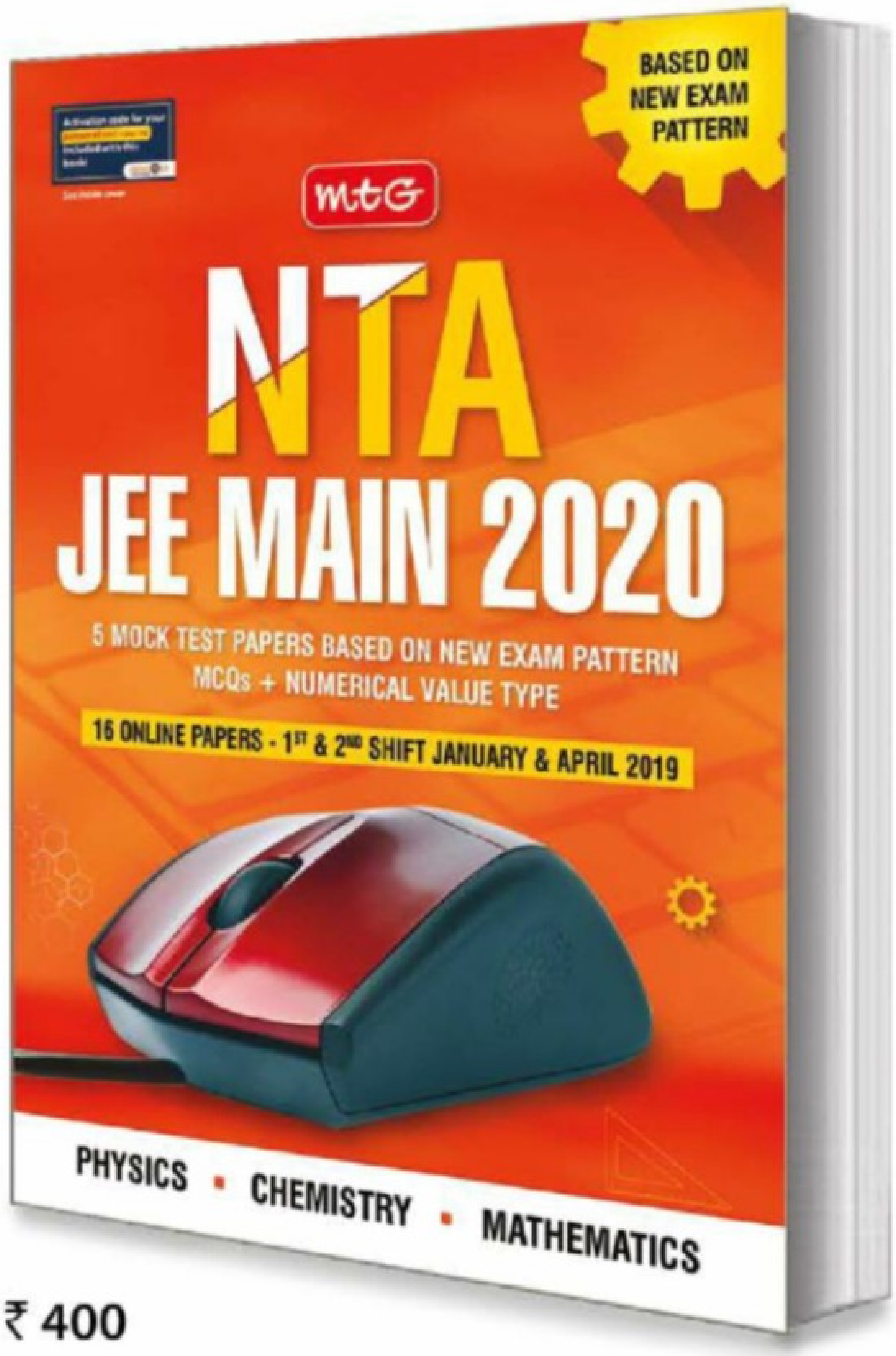
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